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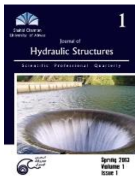

Slope Stability Analysis Using a Self-Adaptive Genetic Algorithm

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Abstract

This paper introduces a methodology for soil slope stability analysis based on optimization, limit equilibrium principles and method of slices. In this study, the slope stability analysis problem is transformed into a constrained nonlinear optimization problem. To solve that, a Self-Adaptive Genetic Algorithm (GA) is utilized. In this study, the slope stability safety factors are the objective functions, slip surface parameters are the decision variables and, the equilibrium equations are the problem constraints. The proposed model satisfies all conditions of the equilibrium completely. It is also applicable to problems with different soil layers, variable soil properties and including pore water pressure. The model is applied against a benchmark example and the results are compared with previous studies. Accordingly, it is found computationally efficient and reliable.

Keywords: Slope stability analysis, self-adaptive GA, Method of slices, Equilibrium analysis, Safety factor

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1. Introduction

Slope stability analysis is of geotechnical engineering problems that has received considerable attention from researchers worldwide. Equilibrium analyses of slope stability are widely used in design of excavation and embankment slopes. There exist a lot of successful applications and experiences on the limit equilibrium methods which make it very popular

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through their simplicity to implement and accuracy of results as well. Indeed, the limit equilibrium methods have been the most widely used methods for slope stability analysis [1]. These methods, in general, satisfy the force and moment equilibrium; boundary conditions and the failure criterion along the slip surface. In context of the limit equilibrium methods, methods of slices are extensively used to cope with complex slope geometries, variable soil properties and the existence of pore water pressure.

Reviewing the literature, the slope stability methods can be categorized in two major groups consisting of the numerical methods, mostly the finite element method [2-4] and analytical methods, mostly based on the methods of slices. The latter encompasses; the ordinary method (1936) [5], simplified Bishop method (1955) [6], simplified Janbu method (1956) [7], Corps of Engineers method (1967) [8], Spencer method (1967) [9], Morgenstern-Price method (1965) [10], Samani and Meidani (2003) [11]. These methods are somehow different in defining the safety factor equations. They also use different assumptions to derive the governing equations and carrying out the stability analyzes as summarized in Table 1.

There are two kinds of solutions for the problem. The first is a simplified solution where the

Table 1. Different methods of slices for slope stability analysis

Method	Assumptions	Equations used	Slip surface
Ordinary method of slices (1936)	• Resultant of side forces (Ei) is parallel to the base of the slice	• Overall moment	Circular
Bishop (1955)	• Resultant of side forces is horizontal	• Overall moment • Vertical forces	Circular
Janbu(1956)	• Location of side force resultants on the sides of the slice (location can be varied) • Uses a correction factor f_o To account for the effect of the inter-slice shear forces.	• Overall moment • Vertical forces • Horizontal forces • Slice moment	Any
Morgenstern and price(1965)	• Inter-slice forces (Xi) related by • V = λ f (x) E form of f (x)	• Overall moment • Vertical forces • Horizontal forces • Slice moment	Any
Spencer(1967)	• Inter-slice forces are parallel	• Overall moment • Vertical forces • Horizontal forces • Slice moment	Any
Samani and Meidani(2003)	• No Assumption	• Overall moment • Vertical forces • Equilibrium of forces in tangential direction to the base of slices	Circular

conditions of static equilibrium are not rigorously satisfied. In this solution, some assumptions are made to obtain the solution in a simple form. The second is a rigorous solution where the equilibrium conditions are completely satisfied [12].

In general, the main features of limit equilibrium methods can be summarized as the following [13]:

1) The sliding body above an assumed slip surface is divided into a number of vertical (or inclined) slices.

2) The strength of the slip surface is mobilized by the same factor of safety, where the cohesion component and the friction component of the strength are reduced equally.

3) Assumptions regarding inter-slice forces are employed to render the problem determinate.

4) The factor of safety is derived from the force or/and moment equilibrium equations.

Table 2. Summary of equations and unknowns associated with limit equilibrium methods

Number of equations	Type of equations
N	Horizontal force equilibrium
N	Vertical force equilibrium
N	Moment equilibrium
N	Mohr-Coulomb failure criterion at the base of slice
$4N$	Total number of equations
Number of unknowns	**Type of unknowns**
N	Total normal force at the base of slice, P_i
N	Shear force at the base of slice, S_i
$N-1$	Inter-slice total normal force, E_i
$N-1$	Inter-slice shear force, X_i
$N-1$	Point of application of the Inter-slice total normal force
N	Point of application of the total normal force at the base of a slice
1	Factor of safety
$6N-2$	Total number of unknowns

Note: N is the number of slices, P_i, S_i, E_i, X_i are introduced in figure 2.

The number of equations and unknowns associated with the limit equilibrium methods are presented in Table 2. It shows that the number of available equilibrium equations is less than the number of unknowns in slope stability problems. As a result, the problem is inherently indeterminate. An indeterminate system of equations has an infinite number of solutions. Using engineering judgment and experiences, one may confine the unknown values between a lower and upper limit in order to manage possible solutions. In this context, the problem could be more systematically treated using the optimization techniques. On this basis, the present study introduces a self-adaptive GA to solve the system of equations of slope stability analysis. The applied procedure satisfies all conditions of equilibrium with a high degree of precision. Using the self-adaptive GA, all constraints of the problem are automatically handled into the optimization with no need for any penalty function on the objective function. For this purpose, a slope stability analyzer model is developed and coupled to the GA. The proposed model is applied against a benchmark example and the results are compared with the other conventional methods.

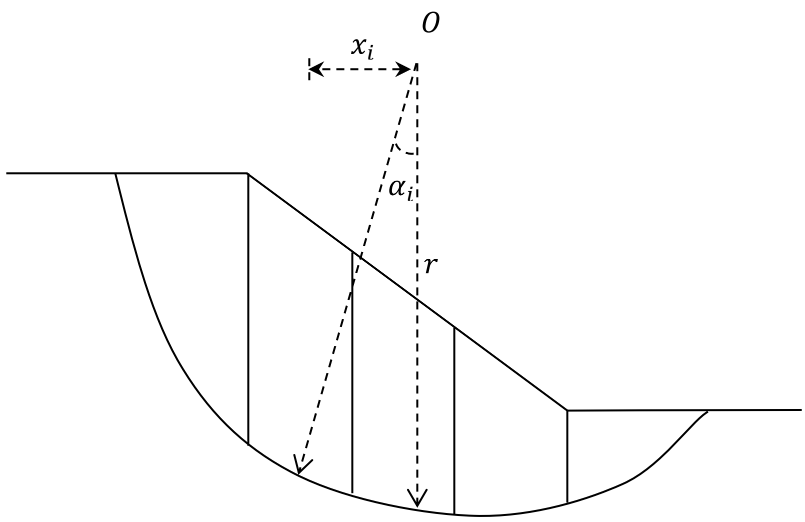

Figure 1. Sliding circular surface subdivided into vertical slices

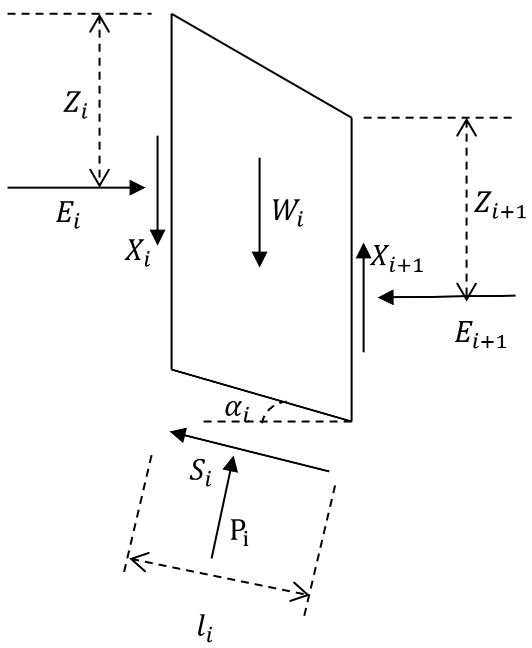

Figure 2. Free body diagram of a slice

2. Governing Equations

Geotechnical engineers frequently use the limit equilibrium methods of analysis when studying slope stability problems. For this purpose, the methods of slices are the most commonly used technique for the sake of their easiness in concept and implementation as well as ability to accommodate complex geometrics and variable soil and water pressure conditions [14].

Fig. 1 shows the potential sliding mass along a trial slip surface through a homogenous slope. The sliding mass is subdivided into a number of vertical slices. The free body diagram of a slice is illustrated in Fig. 2. The forces acting on the slice are its own weight W_i, slide forces, both having shear component X_i, and normal component E_i, and shear resistance S_i and the normal force P_i acting on the base of slice. Equating the moment of weight of the sliding mass with the moment of external forces acting on the slip surface about the center O of the slip circular surface yields:

$$\sum W_i\ .x_i = \sum S_i\ .r \tag{1}$$

where W_i is slice weight, S_i is shear forces in tangential direction to the base of the slice, and x_i and r are shown in Fig. 1. The relation between the shear strength of failure and equilibrium shear stress along the slide surface can be expressed as the following:

$$\tau = \frac{\tau_f}{F} \tag{2}$$

in which, F is the factor of safety and τ_f is the soil shear strength of failure calculated based on the Mohr-Clumb equation:

$$\tau_f = C' + \left(\frac{P_i}{l_i} - u_i\right).\tan\phi' \tag{3}$$

where C' is drained cohesion of the soil, ϕ' is drained internal friction angle, l_i is the slice base length and u_i is the pore water pressure.

Combining equation 2 and 3 gives:

$$\tau = \frac{1}{F}\left[C' + \left(\frac{P_i}{l_i} - u_i\right).\tan\phi'\right] \tag{4}$$

The vertical equilibrium for the slice i gives:

$$W_i + X_i - X_{i+1} = P_i\,.\cos\alpha_i + S_i\,.\sin\alpha_i \tag{5}$$

Rearranging for P_i yields:

$$P_i = (W_i + X_i - X_{i+1})\,.\sec\alpha_i - S_i\,.\tan\alpha_i \tag{6}$$

Substituting the last expression in equation 4 and simplifying the result gives:

$$S_i = \frac{1}{F + \tan\alpha_i\,.\tan\phi'}\{C'.l_i + [(W_i + X_i - X_{i+1}).\sec\alpha_i - u_i\,.l_i].\tan\phi'\} \tag{7}$$

Hence, by substituting the last expression for S_i in equation 1 yields:

$$r.\sum\frac{\{C'.l_i+[(W_i+X_i-X_{i+1}).\sec\alpha_i-u_i.l_i].\tan\phi'\}}{F+\tan\alpha_i.\tan\phi'}=\sum r.W_i.\ \sin\alpha_i \quad (8)$$

The summation of the normal inter-slice forces should also be zero:

$$\sum(E_i-E_{i+1})=0 \quad (9)$$

Resolving the force acting on the slice in a tangential direction to the base of the slice results:

$$S_i=(E_i-E_{i+1}).\cos\alpha_i+(W_i+X_i-X_{i+1}).\sec\alpha_i \quad (10)$$

Therefor:

$$\sum(E_i-E_{i+1})=\sum[S_i.\sec\alpha_i+(W_i+X_i-X_{i+1}).\tan\alpha_i] \quad (11)$$

Insertion of the value of S_i from equation 7 into equation 11 yields:

$$\sum\{\frac{C'.l_i+[(W_i+X_i-X_{i+1}).\sec\alpha_i-u_i.l_i].\tan\phi'}{F+\tan\alpha_i.\tan\phi'}.\sec\alpha_i-(W_i+X_i-X_{i+1}).\tan\alpha_i\}=0 \quad (12)$$

Equations 8 and 12 are respectively the moment and force equilibrium equations. These equations should be solved to determine the unknowns X_i for every slice and the factor of safety F.

3. The optimization problem

The analysis of slope stability using the limit equilibrium methods is performed in two steps: First, the calculation of the factor of safety for a given slip surface and, second, a search for the critical slip surface with the minimum factor of safety of the slope. As earlier mentioned, the number of equations is less than the number of unknowns and the system of equations is thus indeterminate. Since, the process of finding the critical slip surface is linked to a technique for finding the minimum factor of safety; it could be possible to consider the process as an optimization problem. Here, equation 13 is considered as the optimization objective function. The acceptable bounds for the problem decision variables X_i, xc, yc, and r are considered as the problem constraints on the objective function. By minimizing the objective function subjected to the following inequality constraints, optimum values of the aforementioned decision variables are obtained.

Minimize F (safety factor) Subject to: (13)

$X_{i\_l}\le X_i\le X_{i\_u}\ \ i=1,2,\dots,N$

$x_{c\_l}\le x_c\le x_{c\_u}$

$y_{c\_l}\le y_c\le y_{c\_u}$

$r_l\le r\le r_u$

$(Eq.8)^2+(Eq.12)^2\le\varepsilon$ (14)

where, subscriptions "l" and "u" indicate the lower and upper bounds of decision variables respectively and, ε is an acceptable tolerance to satisfy the compatibility of equations 8 and 12.

4. Self-Adaptive GA

To solve the above constrained mathematical programming model, a self-adaptive GA is developed as the following on the basis of a standard real GA:

1- An initial population with NP chromosomes is randomly generated within range $[0, 1]$. The chromosomes are decoded based on the upper and lower bounds of decision variables. Accordingly, each chromosome contains a set of feasible shear components (X_i) as well as the geometric parameters of slip surface (x_c, y_c, r). The slope stability analyzer program is run against each chromosome and the corresponding objective function (F) and the violation (V) of last constraint equation 14 are evaluated.

2- The binary tournament selection method [15] is used to select the parents. Through this step, the problem constraint (equation 14) is also handled so that, for each parent, two chromosomes x and y are randomly picked up from the population. x wins the tournament if one of the following conditions is met otherwise, y wins.

a. Both x and y are feasible but x has a greater objective function value.
b. Both x and y are infeasible but x has a smaller constraint violation.
c. x is feasible but y is not.

Accordingly, there is no need to penalize the objective function when a chromosome is infeasible. By using the above simple scheme, the GA can freely search into the problem decision space and gradually approach to the feasible regions.

The number of parents is considered to be half of the population size $(\mathrm{NP}/2)$. After all the required parents were selected, they are transferred to the mating pool to generate new offsprings.

3- The blend crossover method (BLX- α) proposed by Eshelman and Shaffer (1993) [16], is applied to each couple in the mating pool resulting in two children. When the crossover operator is applied to all couples, the population of children with NP size is created.

4- A few genes in the new population are mutated.

5- The children population is introduced to the slope stability analyzer program and F and V values in each new chromosome are evaluated.

6- The old and new populations are combined resulting in a population with 2NP size. The combined population is then divided into two subsets with respect to the feasibility and infeasibility of the chromosomes. The chromosomes with zero constraint violation are transferred to the feasible subset and the chromosomes with nonzero constraint violation are transferred to the infeasible subset. Let the size of feasible and infeasible subsets be respectively NF and NI so that, $\mathrm{NF} + \mathrm{NI} = 2\mathrm{NP}$.

7- To form the new generation, we need to select the best NP chromosomes from 2NP chromosomes in the combined population. For this purpose, first the feasible subset is taken into account. If $\mathrm{NF} \geq \mathrm{NP}$, the feasible subset is sorted in descending order of the objective function value F. Then, the top NP chromosomes are selected as the next generation. Otherwise, if $\mathrm{NF} < \mathrm{NP}$, all NF chromosomes in the feasible subset are selected for the next generation. For the remaining $\mathrm{NP} - \mathrm{NF}$ chromosomes, the infeasible subset is sorted in ascending order of the constraint violation value $\mathrm{V_{Rel\_total}}$. Then, the top $\mathrm{NP} - \mathrm{NF}$ chromosomes in this subset are added to the selected feasible chromosomes. It is worth mentioning that, since the parents and children are combined in each generation and the next generation is derived from both, the elitism is automatically preserved in the GA.

8- After the new generation was formed, the algorithm is repeated from step 2 until no further improvement is seen in the objective function.

5. Example

In this section, an illustrative example from the literature is adopted and analyzed using the proposed method. The geometry and soil parameters are presented in Fig 3. It is supposed that the center of the coordinate system is at point A. The factor of safety and slip surface geometric parameters are considered to be unknown.

The example is solved with 10 slices. Upper and lower bounds of decision variables are shown in Table 3. To solve the problem the GA population was decided 50 and the mutation ratio is 0.05. After about 200 generations the best results of the optimization were obtained as the following; $F = 2.35$; $x_c = 4.17\ m$; $y_c = 11.67\ m$ and $r = 9.4\ m$. For more investigations, the safety factors evaluated study in Table 4. Also, Table 5 in the previous studies are compared to the current presents the inter-slice shear forces obtained here it is compared to the previous works. Accordingly, concluded that the model has a good agreement with the previous well-known methods. Furthermore, the maximum constraint violation of $(Eq8)^2 + (Eq12)^2$ is obtained 7E-06 which means that, both moment and force equilibrium equations have been precisely fulfilled through the applied model.

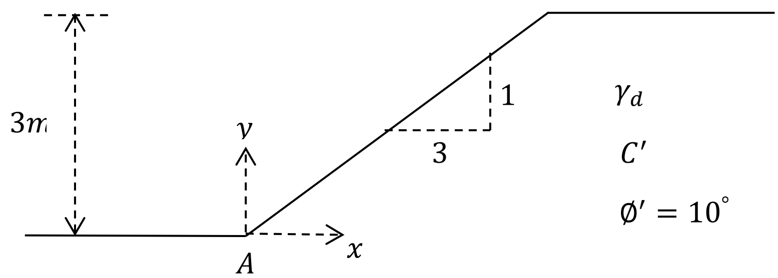

Figure 3. Geometry and soil parameters of example

Table 3. Upper and lower bounds of unknown values

	Lower limit	Upper limit
xc	0	15
yc	0	15
r	3	15
X1	1	10
X2	1	20
X3	1	22
X4	1	22
X5	1	25
X6	1	22
X7	1	20

X8	1	20
X9	1	10

Table 4. Factor of safety calculated by various methods

Bishop (1955)	Janbu (1956)	Morgenstern-Price (1965)	Samani-Meidani (2003)	Self-Adaptive GA (Present study)
2.394	2.125	2.391	2.437	2.350

Table 5. Inter-slice shear force calculated by different methods

	Constant-Interslice	Half-Sine	Corps of	Samani-Meidani	Self-Adaptive GA(Present Study)
X1	3.38	1.39	3.4	3	3.15
X2	7.27	5.49	7.3	7.4	7.84
X3	10.95	11.29	11	12.3	13.16
X4	13.54	16.28	13.6	19.7	20.82
X5	14.44	18.1	14.5	21.8	20.02
X6	14	17.04	14.06	22.5	20.68
X7	10.7	10.92	10.74	22.1	20.21
X8	7.35	6.12	7.4	21.5	19.63
X9	2.44	1.24	2.45	12	11.04

6. Conclusion

Design or evaluation of any embankment and slope to resist the destructive effects safely, requires to solve a complicated problem in the field of geotechnical engineering. The limit equilibrium methods are the most common technique for the slope stability analysis. The present study aimed at introducing an optimization framework based on optimization to solve the slope stability analysis problem. For this purpose, a self-adaptive GA was coupled to the Limit equilibrium and method of slices. Accordingly, the slope stability problem was transformed into a constrained optimization problem. Using the Self-Adaptive GA, there is no need to penalize the objective function when a chromosome violates the problem constraints. Through the proposed scheme, the GA can freely search into the problem decision space and gradually approach to the feasible regions where, the moment and force equilibrium equations are completely met. The method is applied to an example slope. The results for safety factor, inter-slice shear forces, coordinates of the slip circle center and radius were calculated. The results showed that model is in a good agreement with previous studies. The proposed procedure would be also applicable for dealing with problems with deferent soil layers, variable soil properties and having pore water pressure.

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