

Proposing a Discharge Coefficient Equation for Triangular Labyrinth Spillways Based on Laboratory Studies

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Abstract

Labyrinth spillways are considered as suitable and economic structures because, firstly, their discharge flow rate, under low hydraulic heads, is high, and secondly, they occupy less space. The flow over these spillways is three-dimensional and is influenced by several parameters. This study endeavors to offer a new equation for the calculation of the discharge flow of triangular labyrinth spillways by using the Buckingham Theory, Genetic Algorithm, the equations offered by other researchers and non-dimensional laboratory parameters. To do so, several experiments were carried out in the hydraulic laboratory of the Department of water science at BuAli Sina University in Hamedan. These experiments were done on triangular labyrinth spillways in a straight condition in certain flumes which were 10 meters long, 0.83 meters wide, and 0.5 meters height. To verify the recommended equation, the authors used the information related to the labyrinth spillway design of Bartletts Ferry dam in the US and the equation for triangular spillways suggested by previous researcher. The results indicated that the equation recommended in this study is far more accurate than the previous one.

Keywords: Triangular Labyrinth Spillway, Discharge Coefficient, Buckingham Theory, Genetic Algorithm.

Received: 16 March 2017; Accepted: 25 May 2017

1. Introduction

Multidimensional (Labyrinth) spillways are made of several interconnected sides whose crest axis is, unlike spillways with a linear crest whose axis is straight, a broken line, and this is one of the defining characteristics of these spillways. One of the advantages of these spillways, compared with normal ones, is the fact that in a certain width, they have longer crests which

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enables them to provide more discharge (Mehboudi 2015, Crookston 2010). For instance, the labyrinth spillway built over the Beni Badhel dam in Algeria before the Second World War has 12 times the discharge capacity of a standard ogee spillway (Erpicum et al. 2012). Labyrinth spillways are more efficient in low hydraulic heads, and the increase in the head will result in the gradual decrease of their water transmissivity (Taylor 1968). These spillways can have such different shapes as rectangular, triangular and trapezoidal (Crookston 2010). Existing documents suggest that the majority of labyrinth spillways have been built since 1970 (Falvey 2003). An overview of these spillways can be seen in Figure 1.

These multifaceted spillways can be used in different situations and conditions, and this is one of the reasons why they have been developed increasingly. For instance, these spillways can be used to channel huge floods under low hydraulic heads and to increase the capacity of the reservoir storage (Crookston 2010). As an example of their widespread usage, one can refer to the New Mexico Interstate Stream Commission's request of The Bureau of Reclamation to increase the storage capacity of the Ute reservoir in New Mexico by using a gated spillway or any other suitable and economic structure. The Bureau suggested a multifaceted spillway made of several smaller triangular and trapezoidal spillways because they believed that it could increase the length of the canal width (Houston 1982). Another example is the inflow design flood of the Hyrum dam which was reevaluated in 1981. It was discovered that the existing structure could not pass the flood; as a result, it was suggested that an ungated auxiliary labyrinth spillway be built (Houston 1983). Moreover, a labyrinth spillway was recommended for increasing the reservoir storage capacity and the discharge flow of the Prado dam built over the Ana Santa River in Los Angeles (Copeland and Fletcher 2000).

A= Half-length of weir apex
 B= Length of weir side wall
 H= Upstream piezometric head
 H_v = Upstream velocity head
 H_t = Upstream total head
 H_d = Downstream piezometric head
 $L = 4A + 2B$ = Developed length of one cycle
 $\frac{L}{w}$ = Length magnification
 $L_e = nL$ = Total length of weir
 n = Number of weir cycles in plan
 p = Weir height
 w = Width of one cycle of weir
 $\frac{w}{p}$ = Vertical aspect ratio
 $W = nw$ = Total width of spillway
 α = sidewall angle
 $\frac{H_T}{p}$ = Total Head to weir height ratio
 T = Wall thickness

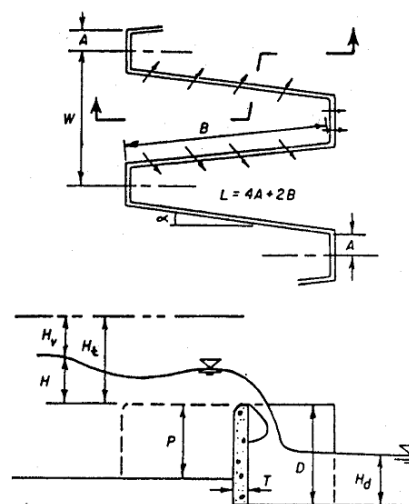


Figure 1. An overview of linear labyrinth spillways with details (Copeland and Fletcher 2000).

The main advantages and implications of these spillways include, but are not limited to the followings: they have the ability to pass flood and adjust upstream flow (Savage 2016), they need fewer ferry boards in comparison with linear spillways (Crookston 2010), they control the

quality of the water via an aerated flow controller (Ghare et al. 2008), they can be used as a tilt breaker structure that can reduce energy (Ghare et al. 2008), they can be used as both service and emergency spillways in reservoir dams where the water level is high, as is the flow rate (Tacail et al. 1990), they can be used to stabilize and protect the depth of the flow (Ercicum et al 2012), their low costs of construction and maintenance compared with gated spillways is also another advantage of labyrinth spillways (Ercicum et al 2012). Nevertheless, one of their weaknesses, compared to linear spillways, is that they are more susceptible to damage and blockage caused by ice and floating objects (Taylor 1968).

The flow that goes through labyrinth spillways is three-dimensional and is therefore quite complex. Due to the fact that this flow is influenced by various factors, several experiments need to be carried out for the determination of influential parameters that impact that discharge flow. The first tests and experiments were carried out by Gentilini in Italy in 1941. He put together several diagonal spillways in a laboratory flume and experimented with different positions for a cutting edge triangular spillway. He compared the spillway discharge coefficient in triangular weirs with those of linear ones in a graph based on the total head ratio to the height of the weir (Crookston 2010). In 1968, Taylor conducted a number of studies on the performance of labyrinth spillways (with 58 models that included mostly triangular but also trapezoidal and rectangular structures with both sharp and half-round crests). They assumed a corresponding relation between the performance of the labyrinth spillways and the performance of linear spillways. They published their results in curved graphs which became the basis for many future studies (Taylor 1968). Darvas (1971) used the results of the studies conducted on the hydraulic models of both Avon and Woronora dams in order to propose an empirical equation for the discharge flow as well as the design diagrams for trapezoidal labyrinth spillways with quarter-round crests that lack an apron (Crookston 2010). Carollo et al (2012) investigated the passing flow over triangular labyrinth spillways with sharp crests as well as the longitudinal magnification 1.41 and 2 (L/w) in laboratory studies. To check the flow rate over the triangular labyrinth spillway, they used the equation put forth by Ghodsian (2009) and found out that the equation needs further calibration by using the Buckingham theory and the Bazin formula so that a new discharge equation is recommended. The Bazin equation works regardless of the approaching velocity in the reservoir. Therefore, they used Piezometric height instead of the total height in their proposed equation. To investigate the precision of this equation, they compared the calculated flow done by this equation with the observed flow in 6 weirs. The margin of error was 3.6-12.9%. Gupta et al (2013) based their study on Khode et al. (2012), and, using multivariable regression analysis, they tried to propose an improved equation for the discharge coefficient that would cover all the angles of the conductor walls between 8 to 30 degrees. Ramakrishnan et al. (2014) carried out laboratory studies on three piano key shaped spillways, i.e. a type of non-linear (labyrinth) spillway with different geometrical shapes, and they were able to propose certain graphs which can be used to determine the discharge coefficient in the face of the ratio of the total head to weir height of $\frac{H_T}{p}$. The laboratory findings of Bilhan et al.

(2016) indicated that trapezoidal labyrinth spillways are superior to linear and circular ones when it comes to the discharge flow rate and their ease of construction.

Most of the researchers have used the tests carried out by themselves or others in order to determine the discharge coefficient. Moreover, in most cases, they have taken into consideration only some of the important factors, for example, they have only focused on the impact of $\frac{H_t}{p}$. These researchers have failed to consider that other factors can also have significant influence on the discharge coefficient. Some of these other factors include: height, thickness, the shape of the crest, the longitudinal magnification ratio, the angle of the side-wall and the main direction of the flow, the length of the apex and the submerging depth of the downstream, etc. (Tacail et al. (1990), Tullis et al. 1995). Due to the fact that it is both time-consuming and costly to consider all of the influential parameters that influence the discharge flow, the present study endeavors to take into account more influential parameters, compared to other studies, and it uses the theory of Buckingham and genetic algorithm to come up with a simple, precise, practical, and common equation that can estimate the discharge flow of labyrinth spillways.

2. Discharge Equation in Labyrinth Spillways

Figure 1 shows a design for a labyrinth spillway. The passing flow through this spillway could be considered as a function of the following parameters.

$$f(Q, A, B, L, n, L_e, w, W, p, D, T, H_t, H_D, g, \mu, \sigma, \text{crest shape}, \alpha) = 0 \quad (1)$$

In this equation, Q is the spillway discharge, A is half the length of the apex (which could be considered zero if it is triangular), B is the length of the crest on the adjacent side, L is the length of the spillway's crest in one cycle, which equals $4A + 2B$, n is the number of cycles, L_e the total effective length of the spillway, which equals nL , w is the width of a cycle in the spillway, W is the total width of the spillway, which equals nw , p is the crest height from the bed of the upstream spillway, D is the crest height from the bed of the downstream spillway, T is the thickness of the spillway's wall, H_t is the total hydraulic head in the upstream, which equals the sum of the hydrostatic head over the (H) spillway and the velocity head (H_v), H_D is the total hydraulic head in the downstream of the spillway, g is gravitational acceleration, μ is water viscosity, σ is the surface tension, and α is the angle formed by the side wall of the spillway and the main flow direction. B , α , W and L_e are dependent parameters; therefore, their impact in equation (1) will be indirect. For instance, if we know what L and w are, we can then determine α . So, we'll have:

$$f(Q, A, L, n, w, p, D, T, H_t, H_D, g, \mu, \sigma, \text{crest shape}) = 0 \quad (2)$$

If the scale of the model and the velocity are chosen carefully, we can then ignore the influence of the surface tension and the viscosity (U. S. Department of Interior 1980).

$$f(Q, A, L, n, w, p, D, T, H_t, H_D, g, \text{crest shape}) = 0 \quad (3)$$

We can write equation 3 in the following way by using dimensional analysis as well as the Buckingham theory, and by keeping in mind that we can combine the non-dimensional parameters (multiplication, addition, and exponentiation) (Novak et al. 2010):

$$f\left(\frac{Q}{Lp^2\sqrt{g}}, \frac{A}{w}, \frac{L}{w}, n, \frac{D}{p}, \frac{T}{p}, \frac{H_t}{p}, \frac{H_D}{p}, \frac{w}{p}, \text{crest shape}\right) = 0 \quad (4)$$

If we divide the first non-dimensional parameter $\frac{Q}{Lp^2\sqrt{g}}$ by the fourth parameter (n), we can write the equation of discharge flow of the labyrinth spillway as:

$$Q = nwH_t^{\frac{3}{2}}\sqrt{g}f\left(\frac{A}{w}, \frac{L}{w}, \frac{D}{p}, \frac{T}{p}, \frac{H_t}{p}, \frac{H_D}{p}, \frac{w}{p}, \text{crest shape}\right) = C_dnwH_t^{\frac{3}{2}}\sqrt{g} = C_dWH_t^{\frac{3}{2}}\sqrt{g} \quad (5)$$

Therefore, the discharge coefficient will be:

$$C_d = f\left(\frac{A}{w}, \frac{L}{w}, \frac{D}{p}, \frac{T}{p}, \frac{H_t}{p}, \frac{H_D}{p}, \frac{w}{p}, \text{crest shape}\right) \quad (6)$$

3. Genetic Algorithm

Genetic Algorithm (GM for short) is derived from Evolutionary Algorithms and Darwin's Natural Selection principle. In fact, the search method is based on optimization algorithms as well as the structure of the genes, chromosomes, and inheritance, and it has three different functions of selection, Crossover, and Mutation. This method was first introduced by Holland (1960) in the University of Michigan and was later expanded and developed by a number of his students (1970) including Goldberg (Coley 1999). In Genetic Algorithms, several answers are generated for a given problem based on the random selection of a few digits in the intended range (fitting). This selection of answers is called the initial population, and each individual answer is called a chromosome. Later, superior chromosomes are selected by the operators of the genetic algorithm and are combined and put together (Crossover). The next stage is mutation which is performed on the combined chromosomes. Finally, the current population is added to the recent population of combined and mutated chromosomes (replacement) (matlabsite.com). The schema of this method is presented in figure 2.

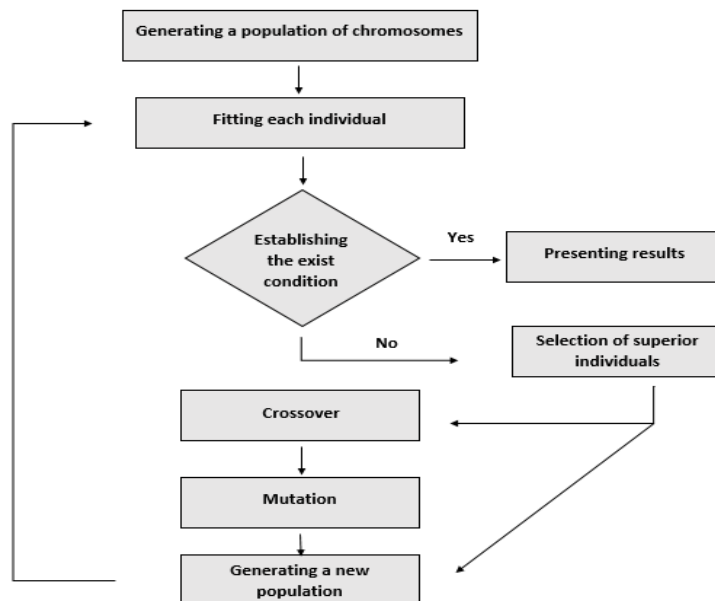


Figure 2. The schema of different stages of the Genetic Algorithm

4. The Objective Functions and Decision Variables

The decision variables in this study include the coefficients of the discharge equations in table 1 as well as the coefficients of the proposed equations (specified later in this paper). The RMSE index could be used to determine the optimum values of these parameters (Fox 1981).

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (O_i - S_i)^2}{n}} \quad (7)$$

In the above equation, O and S are the observed value and the calculated value of discharge flow in the labyrinth spillway, respectively, and n is the number of data. The RMSE index follows the nature of the data, i.e. if O and S, which are observed and calculated values of discharge flow in the labyrinth spillway, are calculated in liters per second, then the results of this equation will also be in liters per second. The RMSE index could be equal to or greater than zero. However, the ideal value for this index is zero.

5. Material and Method

The hydraulic model tests of this study were carried out in the Hydraulic laboratory of the Department of Agriculture at Bu-Ali Sina University in Hamedan. A 10-meter-long glass flume, with 0.83 meters of width and 0.5 meters of height was used in this study (figure 3). The flume was fed by a pump with a flow rate of 58 liters per second, and the discharge flow was controlled with a regulator. An ultrasonic flow meter was used to measure the flow rate; also, a luminometer was used to measure the water level with a precision of ± 0.01 Centimeter. Triangular spillways with quarter-round crests were used in this study. The spillways were built with plexiglass and silicone adhesive was used for seals. An apron was used to strengthen the structure and to make it more stable and resistant against water pressure. Due to the limitations caused by the flume width, and to avoid shrinking the dimensions of the model, and to comply with non-interference conditions of overflow tables (which is necessary for the proper functioning of the labyrinth spillway) the tests were performed in one cycle. Three models were made in total, and except for the difference in their longitudinal magnifications $L/W = 2, 3, 4$, they were identical in their other dimensions. In order to investigate the influence of a lack of levelness or alignment on both sides of the spillway, researchers first fixed the level of the downstream and increased the level of the upstream by using 2.5 to 3- centimeter tiles. They leveled both sides of the spillway once more, and then keeping the level of the upstream at a fixed point, they decreased the level of the downstream, using the above-mentioned method (it should be noted one more time that adhesive silicon was used to seal the sides and seams). In general, the tests were performed and the results of the tests were 1, 1.17, 1.36, 1.44, 1.45, 1.5, 1.72, 1.8, 1.82, 2.22, 2.57, and 4 for $\frac{D}{p}$, and 1.69, 1.98, 2.3, 54.44, 2.4, 2.92, 3, and 4.35 for $\frac{w}{p}$, and 0.057, 0.067, 0.078, 0.082, 0.086, 0.098, 0.103, 0.147, 0.229 for $\frac{T}{p}$. Due to the huge number of non-dimensional parameters, and to determine the equation for the discharge coefficient and its interrelations, 200 tests were performed for the discharge flow rates of 1 to 45 liters per second. Figure 3 shows an overview of the laboratory flume used in this study.

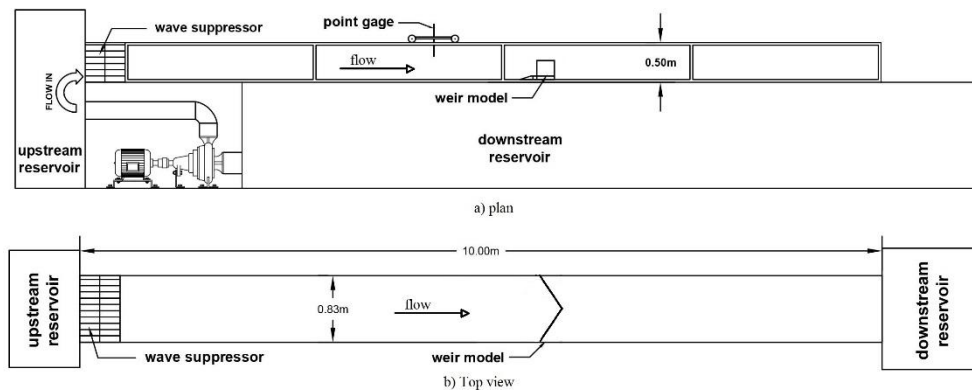


Figure 3. An overview of the laboratory flume used in this study.

The tests were carried out as follows. After a stable and smooth flow was established, the water level in the upstream (the hydrostatic head) was measured by a laminator from a distance where the current was not yet impacted by the spillway. Having established the hydrostatic head in the upstream as well as the height of the spillway, the hydrostatic head in the downstream was also determined. On the other hand, having known the hydrostatic head and cross-sectional area of the canal, the researchers were also able to determine the corresponding velocity head in the upstream of the spillway. Other conditions of this experiment include: the design of labyrinth spillways is usually done in a way that prevents submerging currents in the downstream (Tullis et al. 1995); therefore, these tests were carried out in non-submerging conditions of the spillway. Furthermore, all necessary steps were taken to ignore the interference of overflow tables. The maximum Froude number 0.57 was determined in all of the tests; moreover, Reynolds number is large enough so that we can ignore the effect of viscosity forces. No aerating equipment was used during the tests, and the whole aeration process was done naturally. Finally, it should be noted that the roughness of the floor and the walls of the canal as well as the roughness of the spillway surface were fixed.

6. Results and Discussion

Triangular spillways were chosen for the study of labyrinth spillways ($A=0$) in this research; therefore, it should be noted that the influence of the shape of the spillway on the discharge rate was not analyzed in this study. Moreover, due to the fact that all the experiments and tests were carried out in non-submerging conditions, and that in such conditions, the depth of the downstream current has no impact on the discharge flow (Falvey 2003), the researchers rewrote equation 6 in the following way:

$$C = f\left(\frac{L}{w}, \frac{D}{p}, \frac{T}{p}, \frac{H_t}{p}, \frac{w}{p}\right) \quad (8)$$

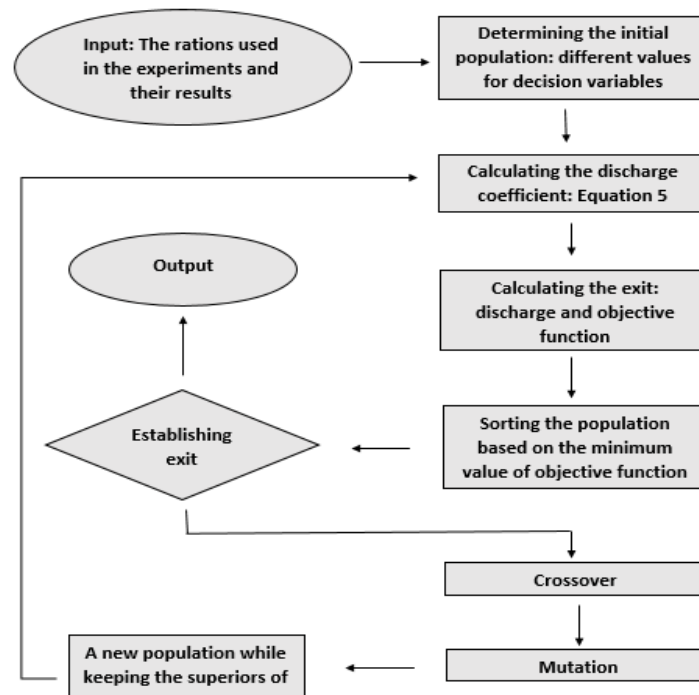
Several equations were analyzed and studied based on previous studies as well as data analysis; consequently, the best equation for the estimation of the discharge flow over the spillway is presented in Table 1. It should be noted that, in order to calculate the discharge flow, this equation should be substituted in equation (5). Moreover, in order to compare the efficiency of the recommended equation, the researchers compared it with the equation suggested for triangular spillways by Ghodsian (2009).

Table 1. Equations for the calculation of the discharge flow rate of triangular labyrinth spillways

No.	Researcher	C_d
1	Author, based on Lux and Hinchliff (1985)'s study	$\frac{1}{k_1 + k_2 \frac{w^{k_7}}{p} + k_3 \frac{H_t^{k_8}}{p} + k_4 \frac{D^{k_9}}{p} + k_5 \frac{L^{k_{10}}}{w} + k_6 \frac{T^{k_{11}}}{p} \frac{w}{p} + k_{12}}$
2	Ghodsian (2009)*	$k_1 \left(\frac{H_t}{p}\right)^{k_2} \left(\frac{L}{w}\right)^{k_3}$

* Ghodsian (2009) used $Q = C_d \sqrt{g} L_e p^{\frac{3}{2}}$ to calculate the discharge flow.

In Table 1, the values of K_i ($i=1, 2, \dots, 17$) are decision variables and are determined through optimization methods and based on the information of the specific spillway. Firstly, the simulation model of the discharge equation (equation 5) was prepared in MATLAB based on each of the discharge coefficient equations in Table 1.

**Figure 4. The schema of the suggested model**

Following that, the optimization model of the genetic Algorithm method was prepared, and then these two models were combined. Figure 4 represents the schema of the final model. In this schema, the values of $\frac{L}{w}, \frac{D}{p}, \frac{T}{p}, \frac{H_t}{p}, \frac{w}{p}, n$ and p as well as the calculated discharge in the tests and the values of the genetic algorithm's operators (with 0.2 mutation rate, 0.5 selection rate, size of the population: 12, and the repetition rate of 1000) were considered as inputs. An initial population was determined for the decision variables. Based on this population and the input values, the discharge coefficient was calculated according to the equation selected from table 3;

then, the discharge flow of the labyrinth spillway was calculated by using one of the discharge equations in table 2. The next stage of the test included the calculation of the target (objective) function (RMSE), based on the selected discharge equation, for the initial population, followed by the arrangement of the population based on the minimum value of the target function. Furthermore, based on the sorted population and using the roulette wheel method, the parents of the next generation are selected. After mutation and crossover, a new population that keeps the superior individuals is selected. The discharge coefficient is determined based on the new population, and this process continues until the difference between the calculated values and the observed values reaches its minimum value. The schema of this process showed in figure 4.

The suggested model determines the coefficients of the discharge equations (Table 1) based on the test results in a way that there is maximum correspondence between the values of the calculated discharge flow and the observed values in the labyrinth spillway. The values of these coefficients and the RMSE for the suggested equation and that of Ghodsian (2009) are presented in Table 2.

Table 2. Optimized coefficients and the RMSE values for the proposed equation model and that of Ghodsian (2009)

Recommended equation for the discharge coefficient	k ₁	k ₂	k ₃	k ₄	k ₅	k ₆	k ₇	k ₈	k ₉	k ₁₀	k ₁₁	k ₁₂	RMSE (lit/s)
Author, based on the study of Lux and Hinchliff (1985)	0.97	-0.19	0.45	-0.55	0.86	-0.32	-0.05	0.7	-0.13	-0.81	0.03	0.46	0.83
Ghodsian (2009)*	0.77	1.17	-0.65										1.45

* Ghodsian (2009) used $Q = C_d \sqrt{g} L_e p^{\frac{3}{2}}$ to calculate the discharge flow.

As it is clear from table 2, the equation proposed by the author works better than the one offered by Ghodsian (2009). Unlike other equations, more non-dimensional parameters have been used in this equation; needless to say, all of these parameters influence the determination of the discharge coefficient of labyrinth spillways. Even though the equation proposed by Ghodsian (2009), i.e. equation 2, has an acceptable level of precision, it is less precise than the proposed equation of this study by 75%.

Figure 5 shows how the objective function changes, and how its value converges towards the minimum value during the process of determining the coefficients of the proposed equation in a run of the Genetic Algorithm. As is clear in figure 5, there is a considerable drop in the mean graph as well as the optimal values of the model; however, after 72 generations, the value becomes fixed and this fixed value moves the best possible value. It should be noted that it is normal for the average values to fluctuate within a certain range because in each generation, the new values, which are the superior values of the current and previous generations, enter a new cycle. Even though they might not have suitable values for the objective function, the average of these values fluctuates within a certain range because there are still superior people (in terms of the minimum value of the target equation) in that cycle. The mean is the average values of each generation, and the best are the values from which the minimum value of the target equation has been derived.

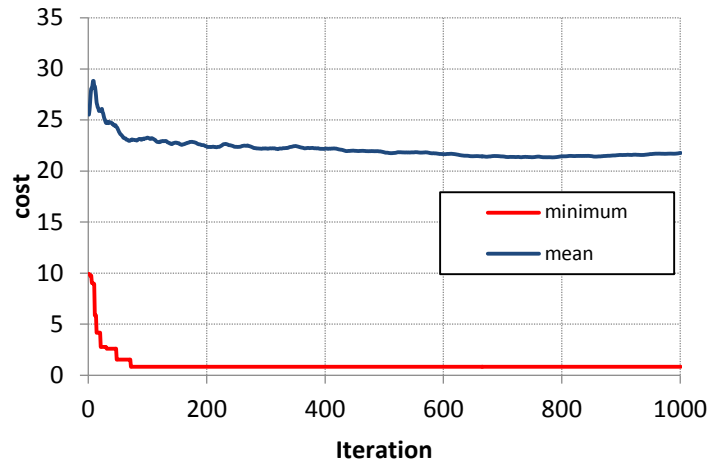


Figure 5. The convergence of the objective function towards the optimized value for the suggested equation

7. Verification

In order to investigate the efficiency of the proposed model, the researchers had to use this model in estimating the discharge flow of a similar labyrinth spillway. The most similar conditions were found to be the labyrinth spillway of the Bartletts Ferry dam in the United States whose tests were carried out in the Department of Civil Engineering at the University of Georgia; the tests had been carried out in a glass flume for a spillway with the following characteristics: $\frac{L}{w} = 4$, $\frac{D}{p} = 1$, $\frac{T}{p} = 0.13$, $\frac{w}{p} = 5.33$ and a quarter-round crest. Unlike triangular spillways, the spillway promontory isn't pointed, but rather it's thin and square-shaped.

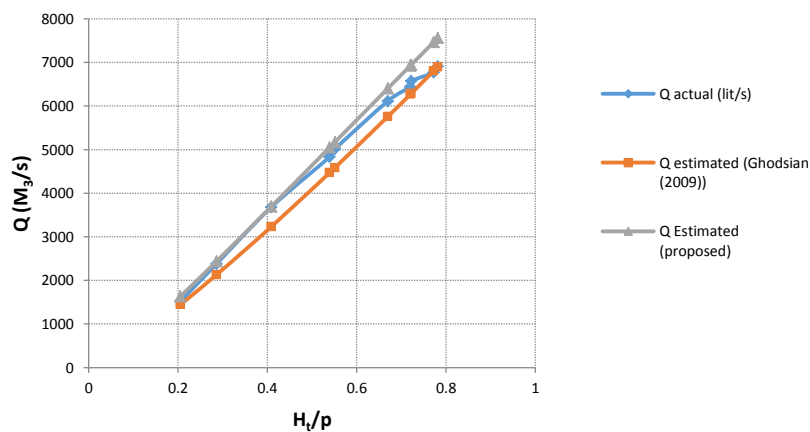


Figure 6. Comparison between the flow of Bartletts Ferry dam in the US in various $\frac{H_t}{p}$ and the calculated flow through the equation proposed by the author and Ghodsian (2009)

Table 3. Comparison between the flow of Bartletts Ferry dame in the US in various $\frac{H_t}{p}$ and the calculated flow through the equation proposed by the author and Ghodsian (2009)

$Q_{act.}$ (lit/s)	$\frac{H_t}{p}$	$Q_{est.1}$ (lit/s)	Error (%)	$Q_{Ghodsian}$ (lit/s)	Error (%)
1520615	0.21	1631983	7.32	1443936	-5.04
2381447	0.29	2437569	2.36	2129675	-10.57
3681190	0.41	3695099	0.38	3232575	-12.19
4833686	0.54	5051534	4.51	4471494	-7.49
5012082	0.55	5171703	3.18	4583742	-8.55
6110776	0.67	6400033	4.73	5754029	-5.84
6442083	0.72	6925119	7.50	6266851	-2.72
6572340	0.72	6943523	5.65	6284959	-4.37
6767726	0.77	7467372	10.34	6804168	0.54
6909311	0.78	7559137	9.41	6895867	-0.19
Mean absolute error (%)			5.54		5.75
Mean absolute error for $\frac{H_t}{D} < 0.7$ (%)			3.75		8.28

According to the findings of the researchers, the difference between the said spillway and the triangular spillway of the present study is negligible. This dam includes 20.5 cycles, and each cycle has a 120-foot crest length on the adjacent side; also, it is 11 feet and 3 inches height, 18 inches thickness and 60 feet long (Mayer & Paul 1980). The results of the calculations for different discharge flows are presented in Figure 6 and Table 3.

According to Table 3 and Figure 6, the proposed equation is even more accurate than the one offered by Ghodsian (2009). According to table 3, the margin of error of the proposed equation for the values of $\frac{H_t}{D} < 0.7$, which means when the flow is not contracted yet, is about 3.75% whereas it is about 8.28% for the equation proposed by Ghodsian (2009). Due to the fact that these spillways are designed for flows before they reach contraction levels, $\frac{H_t}{D} < 0.7$ (Falvey 2003), one can argue that the proposed equation is more precise (about 121%) than Ghodsian's (2009) equation when the water is not contracted, whereas when the water does reach contraction levels, Ghodsian (2009) offers a more accurate equation; this can be seen in Figure 6. Indeed, it should be noted that finding errors in experimental equations is normal and even acceptable; therefore, both equations are capable of estimating the discharge flow coefficients of both triangular and trapezoidal spillways with small promontory. The estimated discharge flow values of the proposed equation is, in a majority of cases, too realistic whereas Ghodsian's (2009) equation estimates the flow to be lower than the observed value, and this is why the estimations of the proposed equation can be used for more accurate and certain results. The following parameters could have resulted in the difference in observed and estimated values: the models of the Bartletts Ferry dam were square-shaped whereas the model used in this study was triangular; also, the conditions of the approaching flow are different in the reservoir and the flume.

8. Conclusion

The present study, endeavored to propose a new equation for the discharge coefficient of triangular labyrinth spillways based on previous experiments, the researches of several scholars of the field, the Genetic Algorithm as well as the dimensional analysis. Furthermore, the author used the equation offered by Ghodsian (2009) in order to investigate the efficiency of the proposed model in comparison with the other equations. And to verify whether or not the proposed equation works properly, the discharge flow estimated by the proposed equation and that of Ghodsian (2009) were compared with the observed values of the Bartletts Ferry dam in the United States. The findings suggest that, compared to Ghodsian's (2009) equation, the proposed model is more precise in general conditions (about 4%) and even more precise (about 121%) when the flow is not contracted yet. Other findings of this study include,

- The equation for the discharge flow of labyrinth spillways could be arrived at by analyzing the information of the specific model of the study as well as dimensional analysis.
- Unlike previous studies that considered the discharge coefficient as a function of one or several non-dimensional parameters, this study proved that the discharge coefficient is influenced by such different parameters as: the longitudinal magnification ratio $\frac{l}{w}$, the ratio of the crest height from the bed of the downstream to the crest height from the bed of the upstream in the spillway $\frac{D}{p}$, the ratio of the thickness of the spillway walls to the crest height from the bed of the upstream $\frac{T}{p}$, the ratio of the total hydraulic head in the upstream to the crest height from the bed of the upstream $\frac{H_T}{p}$, the ratio of the width of one cycle of the spillway to the crest height from the bed of the upstream $\frac{w}{p}$, etc.
- Because discharge flow is a function of several non-dimensional parameters, we can propose various equations for the discharge coefficient and the discharge flow. Through analyzing these parameters and based on the studies of Lux and Hinchliff (1985), the equation which had more precision was proposed alongside the one offered by Ghodsian (2009). Then, the results of these equations were compared. The results indicate that the equations proposed in this study work better than the one proposed by Ghodsian (2009).
- It is important to take into account a number of other parameters which are usually neglected. These parameters include: the shape of the spillway, head losses drops, the conditions of the approaching current, submergence, scouring, etc.
- The optimized coefficients for the proposed discharge equations were derived from test data. Therefore, it is better to determine these coefficients based on the information about any specific spillway.

9. Reference

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