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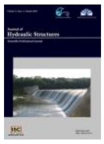

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Development of a Direct Explicit Equation for Hydraulic Design of Semi-elliptical Channels

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Abstract

One of the common concrete channels in irrigation networks is the semielliptical prefabricated channels. Manning formula is usually used to design these channels. The cross-sectional area and the wetted perimeter are required to be calculated in Manning formula. There are no analytical solutions to directly compute these parameters. Thus, numerical integration methods are inevitably used. In this paper, a wide number of various semielliptical channels are regarded and their cross-sectional areas and wetted perimeters for different depths were computed numerically to produce databases for three-dimensional curve fitting. Direct relationships for the cross-sectional area and the wetted perimeter in terms of the channels size and the hydraulic parameters were developed. These relationships were used in the design of the semielliptical concrete channels and the results were compared with the numerical ones. The results were quite close to each other.

Keywords: semielliptical, Manning formula, Numerical method, correlation coefficient

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1. Introduction

Now a day, prefabricated concrete channels are widely used in irrigation networks. These channels are constructed in various shapes. This includes traditional sections such as trapezoidal ones [1, 2, 3, 4] and parabolic sections [5, 6, 7]. One of the other channel sections used is the elliptic sides with horizontal bottoms [8]. The semielliptical concrete channels have been

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recently widely utilized in irrigation projects. They have the advantages of reducing of the constructional cost and land acquisition, reducing evaporation by having smaller water surface widths, reducing sedimentation and they can be constructed fast [9, 10, 11, 12]. There are no explicit direct solutions for the semielliptical cross sections and the wetted perimeters to be used in Manning formula. Vatankhah [12], used a numerical method to calculate the integrals of the cross-section and the wetted perimeter.

In this research, explicit direct relationships for the cross-sectional area and the wetted perimeter of the semielliptical channels are found by using three-dimensional regression analysis. Then, they were used in Manning formula for design purposes.

2. Design Method

Manning formula is used for the design of the semielliptical channels as follows:

$$Q = \frac{1}{n} R^{2/3} A S_0^{1/2} \tag{1}$$

Where:

Q = Flow discharge

n = Manning coefficient

R = Hydraulic radius which is equal to the ratio of the flow cross-sectional area, A, to the wetted perimeter, P

S_0 = Channel bed slope

The flow discharge, Manning coefficient, and the channel bed slope are known. There are many prefabricated concrete semielliptical channels available in the market. The economic optimum design of the semielliptical channel should have the capacity to carry the flow with a limited freeboard. A computer program is developed in this regard in which all the prefabricated available channel sizes are used in the calculation and then the smallest size that is able to pass the flow properly is chosen. The integrals of the cross-sectional areas and the wetted perimeters are calculated by using the trapezoidal rule.

3. Calculation of Flow Cross Sectional Area and Wetted-Perimeter

Figure 1 depicts a typical semielliptical cross-section in which ***a*** and ***b*** are the horizontal and vertical radii of the ellipse, respectively. The cross-section is symmetrical with respect to the **y**-axis. Hence, it is possible to calculate half of the total cross section and then multiplies that by 2 to obtain the total cross-sectional area. The half cross-section is divided into several elements as shown in the figure. Each element is assumed to be trapezoidal. The coordinates of the intersections of the vertical lines of the elements with the elliptical curve of the channel wall can be determined by the equation:

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1 \tag{2}$$

In which x_0 and y_0 are the coordinates of the ellipse center. It is clear that the summation of the trapezoids areas will produce the area of the half of the total flow cross sectional area.

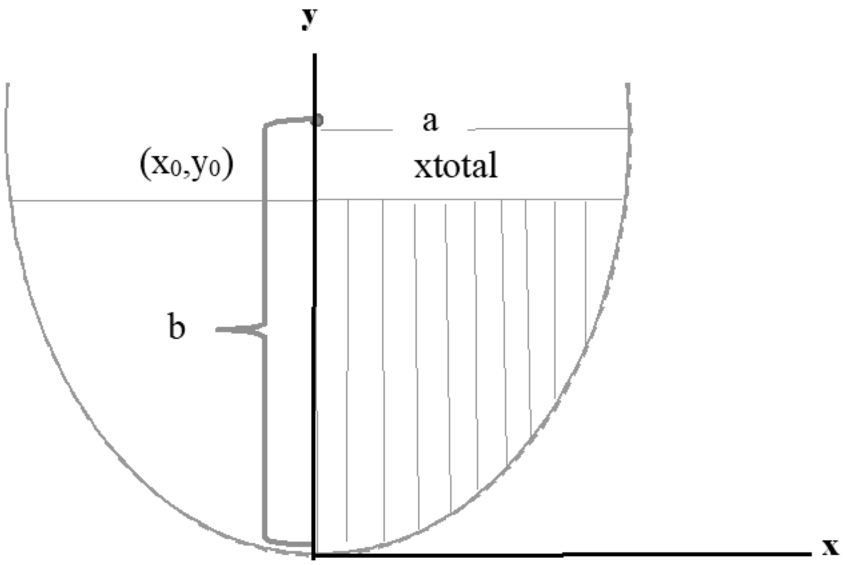

Figure 1. Flow Cross Sectional Area in the Semi-elliptical Channel

The wetted perimeter of each element depicted in Figure 1 can be calculated by the following equation:

$$\Delta L_i = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \quad (3)$$

In which (x_i, y_i) and (x_{i+1}, y_{i+1}) are the coordinates of two sequential points of the ellipse shown in Figure 1. The summation of ΔL_i s results in half of the total wetted perimeter. As an example, semielliptical prefabricated concrete types are considered for the design of a channel to carry out 500 L/s and the bed slope is 0.0005. Manning coefficient is regarded as 0.015. The results of the developed computer program are introduced in Table 1.

Table1. Semielliptical channel types for the given example

Type	Depth (cm)	*b*(cm)	*a*(cm)	n	Q(L/s)	Qcalc(L/s)	Bed Slope
70	26.4	33.4	27.7	0.015	500	15.1609	0.0005
80	28.7	35.7	29.6	0.015	500	18.6132	0.0005
100	32.9	39.9	32.9	0.015	500	25.8852	0.0005
120	36.7	43.7	36	0.015	500	33.9412	0.0005
135	39.4	46.4	38.1	0.015	500	40.3424	0.0005
150	41.9	48.9	40	0.015	500	46.8109	0.0005
180	44.6	51.6	45.4	0.015	500	61.1335	0.0005
200	45.6	52.6	49.4	0.015	500	71.2627	0.0005
230	48.8	55.8	53.5	0.015	500	87.2413	0.0005
250	48.9	55.9	58	0.015	500	98.5231	0.0005
280	50.5	57.5	63	0.015	500	115.764	0.0005
315	52.3	59.3	69	0.015	500	138.064	0.0005

350	53.9	60.9	74.6	0.015	500	160.501	0.0005
400	55.6	62.6	82.8	0.015	500	193.766	0.0005
450	56.1	63.1	92.2	0.015	500	227.925	0.0005
520	56.5	63.5	105.6	0.015	500	277.212	0.0005
600	58.2	65.2	118.7	0.015	500	337.818	0.0005
700	62.8	69.8	135.6	0.015	500	447.656	0.0005
800	**64.62604954**	74.4	144.7	0.015	500	500	0.0005
900	**62.74162577**	78.7	153.4	0.015	500	500	0.0005
1000	61.23847894	82.8	161.6	0.015	500	500	0.0005

The first column of Table 1 represents the channel type and the calculated flow depths are given in the second column. The third and fourth columns represent the vertical and horizontal radii, respectively. The seventh column represents the calculated flow corresponds to the channel size types. The results show that channel types 70 to 700 are too small to pass the given flow and types 800, 900, and 1000 are able to pass the flow. It is clear that type 800 is the economical one which should be regarded as a final design.

4. Explicit Relationships for the Flow Cross Sectional Area and the Wetted Perimeter

In order to develop explicit direct relationships for the flow cross-sectional area and the wetted perimeter, a large number of condition combinations of semielliptical channels with various hydraulic conditions are considered. The developed computer program is applied to all these combinations similar to that which is shown in Table 1. Based on the obtained results the dimensionless flow cross-sectional area A/ab is regarded as a function of the dimensionless variables y/a, y/b and the least squares method was used to obtain the best fitted function. This task was done by the Table-curve 3D software. The results indicated that A/ab is only a function of y/b and the best-fitted function is obtained as:

$$\frac{A}{ab}=e^{0.552486-0.111776(y/b)^2+1.46526Ln(y/b)} \tag{4}$$

The correlation coefficient for the above function is calculated as $r^2=0.99999994$. Similarly, the dimensionless wetted perimeter $L/\sqrt{a^2+b^2}$ is considered as a function of the dimensionless variables y/a, y/b and the best-fitted function is obtained as:

$$\frac{L}{\sqrt{a^2+b^2}}=\frac{0.413705}{\sqrt{y/a}}+3.2257358\sqrt{y/b}-1.3992385 \tag{5}$$

The correlation coefficient for the function given in Equation (5) is calculated as $r^2=0.99999994$.

Substituting the calculated functions for the flow cross-sectional area and the wetted perimeter given by Equations (4) and (5), respectively, in Manning equation results in:

$$Q=\frac{1}{n}\left(\frac{abe^{0.552486-0.111776(y/b)^2+1.46526Ln(y/b)}}{\sqrt{a^2+b^2}\left(\frac{0.413705}{\sqrt{y/a}}+3.2257358\sqrt{y/b}-1.3992385\right)}\right)^{2/3} abe^{0.552486-0.111776(y/b)^2+1.46526Ln(y/b)}\sqrt{S_0} \quad (6)$$

The developed equation (6) can be used simply for the design of semielliptical channels.

5. Evaluation of the Developed Equation

Various semielliptical channels with different slopes are considered and the flow discharges are calculated by two methods, the developed equation [Equation (6)] and the numerical trapezoidal method. The results are given in Table 2. Column 7 represents the difference percent between the developed and numerical methods. It is noted that the differences are very small which proves that the developed equation has the desired accuracy.

Table2. Percent of differences Between the Developed Equation and The Numerical Method

Depth (cm)	*a*	*b*	Bed Slope *S*	*n*	*Q* Eq.(6)	Q(Numerical)	Error %
46.23724	0.315	0.575	0.0005	0.015	99.9	99.99	0.10001
43.25029	0.345	0.593	0.0005	0.015	99.59	99.99	0.414032
51.95147	0.461	0.631	0.0005	0.015	200.17	200	0.08504
46.93797	0.528	0.635	0.0005	0.015	199.86	200	0.067927
54.36399	0.5935	0.652	0.0005	0.015	299.2	300	0.268229
58.88418	0.678	0.698	0.0005	0.015	398.07	400.00	0.484581
64.62605	0.7235	0.744	0.0005	0.015	497.41	500.00	0.520581
62.74163	0.767	0.787	0.0005	0.015	497.97	500	0.406979
69.53065	0.767	0.787	0.0005	0.015	596.76	600	0.542812
67.69017	0.808	0.828	0.0005	0.015	597.29	600	0.454458
46.67753	0.29	0.559	0.0006	0.015	100.05	100	0.05304
43.82707	0.315	0.575	0.0006	0.015	99.65	100	0.350047
53.65199	0.414	0.626	0.0006	0.015	200.52	200	0.258765
49.30414	0.461	0.631	0.0006	0.015	199.98	200	0.01005
56.09775	0.528	0.635	0.0006	0.015	300.01	300	0.003339
51.65093	0.5935	0.652	0.0006	0.015	299.32	300	0.22821
55.95632	0.678	0.698	0.0006	0.015	398.38	400	0.407195
61.38002	0.7235	0.744	0.0006	0.015	497.72	500	0.457451
66.01671	0.767	0.787	0.0006	0.015	597.08	600	0.488993
70.12686	0.808	0.828	0.0006	0.015	696.47	700	0.506404
75.71512	0.808	0.828	0.0006	0.015	795.54	800	0.560142

6. Summary and Conclusions

A new explicit equation for the hydraulic design of the semielliptical channels has been developed in this research. In this regard, a computer program has been initially prepared in which the flow discharge is calculated by the Manning formula and the flow cross-sectional area and the wetted perimeters are calculated by using the trapezoidal numerical integration method. A large number of the database include the flow discharges against the channel characteristics were provided by the developed computer program. The database was converted to dimensionless parameters for the cross-sectional area and the wetted perimeter versus other dimensionless hydraulic parameters and then the Table-curve 3D software was utilized to obtain the best functions which fit the data using the least square method. As a result, explicit direct equations were obtained whose correlation coefficients are equal to almost unity which indicate that the fitted functions have desired accuracies. The accuracy of the developed final equation of the Manning formula was tested by comparing its results with those obtained by the numerical integration method. The results indicate close agreement with each other which shows that the developed explicit direct Manning equation can be used for the purpose of semielliptical channel design with a proper accuracy.

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