Development of A new Hydraulic Relationship for Submerged Slide Gates

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Abstract
Slide gates are one of the most common gates used in hydraulic structures. These gates are often in practice hydraulically submerged. The usual method to determine the flow passes through the gate is solving simultaneously the energy and the momentum nonlinear equations by a numerical method. In this manuscript, a large number of various cases of gate sizes and flows are considered and solved numerically by using these two equations to obtain a database. This database is converted to dimensionless groups and then a direct relationship is developed by using three-dimensional curve fitting. The results obtained by the developed relationship are compared with those obtained by the numerical common method. The comparison results in a good agreement which indicates that the developed relationship is accurate and simple to be used in the design of slide gates.

Keywords: Slide gate, Numerical method, Submerged gate, Curve fitting.

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1. Introduction
Slide gates are common in irrigation networks, used for controlling the flow in channels. The flow passes the gate may result in either a free or submerged hydraulic jump when interacts with the tailwater. The outflow is said to be free when the issuing jet of the supercritical flow is open to the atmosphere and is not submerged. The outflow is known as submerged, if the tailwater

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depth, \( y_3 \) is greater than the conjugate depth needed to form a hydraulic jump. The commonly observed case is the submerged one. Figures 1 and 2 show the free and submerged hydraulic jumps, respectively.

Henry [1] conducted experiments on free and submerged slide gates and showed the experimental results in terms of the gate flow coefficient, \( C_d \) as a function of \( \frac{y_1}{y_G} \) and \( \frac{y_3}{y_G} \). Rao and Rajaratnam [2] also showed a good accord between theory and experiment for the case of submerged flow from a culvert which is similar to the case of the submerged flow from the slide gate.

Rajaratram and Subramaniya [3] have developed the following equation for gates in which free hydraulic jumps occur:

\[
Q = C_d b y_G \sqrt{2g y_1}
\]  

(1)

In which \( Q \) is the flow discharge passes through the gate, \( C_d \) is the gate flow coefficient, \( b \) is the gate width, \( y_G \) is the gate opening, \( y_1 \) is the flow depth upstream the gate, \( y_2 \) is the contracted jet depth going out the gate opening, \( y \) is the water depth just behind the gate, and \( y_3 \) is the tailwater depth. Rajaratnam [4] proved that the gate flow coefficient is a function of \( y_G / y_1 \). Garbrecht [5] developed Equation (2) for the gate flow coefficient, \( C_d \). Henry [1]
proved that \( C_d \) has the form shown in Equation (3). Swamee [6] obtained Equation (4) for the coefficient \( C_d \).

\[
C_d = 0.6468 - 0.1641 \sqrt{\frac{y_G}{y}}
\]  \hspace{1cm} (1)

\[
C_d = \frac{0.6108}{\sqrt{1 + 0.6108 \frac{y_G}{y}}}
\]  \hspace{1cm} (2)

\[
C_d = 0.611 \left( \frac{y_1 - y_G}{y_1 + 0.15 y_G} \right)^{0.072}
\]  \hspace{1cm} (3)

Swamee [7] in his research has shown that the flow coefficient in submerged gates depends on the tailwater depth in addition to the gate opening and the flow depth upstream of the gate. Ferro [8] and Negam et al., [9] et al. studied the case of the simultaneous flow over and under the gate. Negam [10] developed equations to determine the combined flow over a weir and under the gate.

No direct explicit equation has been developed to calculate the flow through submerged gates. In this research, a new direct explicit relationship to calculate the flow in terms of the gate dimensions, upstream flow depth, gate opening, tailwater depth is developed.

2. Materials and methods

2.1. Traditional common method

The flow discharges pass through the gate can be found by the traditional common method in which the energy equation between upstream and cross section 2 and the momentum equation between this cross-section and the tailwater depicted in Figure 2 are solved simultaneously. The energy and momentum equations are given in Equations (5) and (6), respectively:

\[
y_1 + \frac{Q^2}{2g(b_y)} = y + \frac{Q^2}{2g(b_y)}
\]  \hspace{1cm} (5)

in which \( y_2 = 0.61 y_G \)

\[
\frac{y}{2} (by) + \frac{Q^2}{g(b_y)} = y_3 (by_3) + \frac{Q^2}{g(by_3)}
\]  \hspace{1cm} (6)

The nonlinear Equations (5) and (6) are solved by using the solver in Excel program to obtain the unknowns \( Q \) and \( y \).

A large number of cases of submerged gates including various gate dimensions, upstream flow depths, gate openings, and tailwater depths are regarded and solved as indicated above to produce a database covering almost all possible cases in practice. The produced database covers the followings:
\( Q = 0.05 - 100 \text{ m}^3/\text{s} \)
\( y_1 = 0.05 - 10 \text{ m} \)
\( y_3 = 0.04 - 9.7 \text{ m} \)
\( b = 0.5 - 10 \text{ m} \)

Some of the results by the above-explained traditional method are given in Table 1.

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<th>( y_2 (\text{m}) )</th>
<th>( y (\text{m}) )</th>
<th>( Q (\text{m}^3/\text{s}) )</th>
<th>( y_3 (\text{m}) )</th>
<th>( y_0 (\text{m}) )</th>
<th>( b (\text{m}) )</th>
<th>( y_1 (\text{m}) )</th>
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### 2.2 The developed equation using non-linear optimization

The involved variables in the problem are converted to dimensionless groups by using dimensional analysis. The resulted dimensionless groups are as follows:

\[
\frac{y_G}{y_1}, \quad \frac{y_3}{y_1}, \quad \text{and} \quad \frac{Q^2 b}{g (by_G)^3}
\]

\( Q^2 b \) \( g (by_G)^3 \) is considered as a function of \( \frac{y_G}{y_1} \) and \( \frac{y_3}{y_1} \). Many types of functions with unknown coefficients are regarded and by using the Generalized Reduced Gradient (GRG) nonlinear optimization method, the function which results in the best fit in which the summation of the squares of the differences between the obtained function and the given points is minimized. The type of the function which results in the maximum correlation coefficient was as given in the following:

\[
\frac{Q^2 B}{g (by_G)^3} = -10.4 + 2.31 \ln \left( \frac{y_1}{y} \right) + 11.37 \left( \ln \left( \frac{y_1}{y} \right) \right)^2 - 11.03 \left( \ln \left( \frac{y_1}{y} \right) \right)^3 - 21.16 \left( \frac{y_0}{y} \right) - 0.184 \left( \ln \left( \frac{y_0}{y} \right) \right)^2 + 9.81 \left( \ln \left( \frac{y_0}{y} \right) \right) + 17.92 \left( \frac{y_0}{y} \right) + 9.51 \left( \frac{y_0}{y} \right)^2
\]

(7)

The correlation coefficient, \( r^2 \), of the function given in Equation (7) is equal to 0.9997 which indicates that the developed equation is accurate enough to be used for the design of the
submerged slide gates.

3. Results and conclusions

The results of the developed relationship given in Equation (7) are compared with those obtained by the traditional method in which the energy and momentum equation are solved simultaneously in Table 2 for a wide range of gate dimensions and hydraulic conditions.

Table 2. Comparison of the discharges obtained by the traditional method and Equation (7)

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<th>$y_1 (m)$</th>
<th>$b (m)$</th>
<th>$y_G (m)$</th>
<th>$y_3 (m)$</th>
<th>$Q$ by traditional method (m$^3$/s)</th>
<th>$Q$ by Equation (7) (m$^3$/s)</th>
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Referring to Table 2, the results of the comparison show that the discharges calculated by the traditional method and proposed method (Equation (7)) are very close to each other. The developed equation is an explicit equation which can be used directly to calculate the flow discharge through submerged slide gates in a much easier manner compared to the traditional method in which two nonlinear equations must be solved simultaneously. Hence, the developed equation can be used in the design of submerged slide gates directly due to its simplicity and high accuracy.

References


