Using composite ranking to select the most appropriate Multi-Criteria Decision-Making (MCDM) method in the optimal operation of the Dam reservoir

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Abstract

In this study, the performance of the algorithms of Whale (WOA), Differential Evolution (DE), Crow Search (CSA), and Gray Wolf optimization (GWO) were evaluated to operate the Golestan Dam reservoir with the objective function of meeting downstream water demands. After defining the objective function and its constraints, the performance of the algorithms was compared with each other and with the absolute optimal values obtained by GAMS nonlinear programming method (19.41). These values together with each algorithm optimization results were ranked using six multi-criteria decision-making methods of TOPSIS, VIKOR, LINMAP, CODAS, ELECTRE I and Simple Additive Weighting after obtaining the performance evaluation indices of each algorithm (Reliability, reversibility, and vulnerability). Finally, integration methods (Mean, Borda, and Copeland techniques) were used to evaluate the performance of models. The results showed that the average responses of the GWO, WOA, DE, and CSA were 1.08, 1.49, 1.29 and 1.19 times the absolute optimal response and the answers’ coefficient of variation obtained by GWO was 2.113 and 1.43 times smaller than the WOA, DE, and CSA, respectively. Moreover, all integration techniques indicated the superiority of the GWO. Then, the CSA, DE, and WOA algorithms were ranked second to fourth, respectively. On the other hand, the use of these methods in solving the problem of Golestan Dam reservoir optimization was considered appropriate due to the similarity of the results obtained from the integration techniques with the results of TOPSIS, VIKOR and LINMAP methods.

Keywords: Optimal use of dam reservoir, Whale optimization algorithm, Differential Evolution optimization algorithm, Crow search optimization algorithm, Gray wolf optimization algorithm.

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1. Introduction

Multi-criteria decision-making models include a set of goals, criteria, alternatives and decision variables, and the central element of this structure is a decision matrix. This matrix represents the results of the decision for a set of evaluation alternatives and criteria. Due to the existence of different management scenarios, the use of multi-criteria decision models (MCDM) is necessary for optimal decision making. Decision-making in water resources has always been complex due to the need to take into account technical, economic, social and environmental factors, and therefore the use of multi-criteria decision-making models can reduce this complexity to some extent. Therefore, many studies have been done in this regard. Donyaii et al. [1], as well as Wang et al. [2] proposed functions for the optimization model while using multi-criteria decision models to optimize the dam reservoir. Comparing the Simple Additive Weighting (SAW) with other methods, Chang and Yeh [3] found that the simple weighting model method can be considered suitable because it is simple and has similar results with other models. Donyaii et al. [4] after introducing the whale multi-objective optimization algorithm, they evaluated its performance as optimal operation of the Boostan dam reservoir based on the Game Theory Method of Kalai & Smorodinsky. The optimization results showed better performance of the whale multi-objective algorithm than NSGA-II, in both objective functions, as well as the operation policies of Boostan Dam Reservoir have got a very good agreement with the whale multi-objective algorithm as the result of Kalai & Smorodinsky method of Game Theory. Afkhamifar & Sarraf [5] evaluated the performance of two models of Extreme Learning Machines (ELM), Artificial Neural Network (ANN) and the combination of two models with wavelet transmission algorithms (W-ELM and W-ANN). They revealed that the hybrid model of W-ELM-QPSO has a better performance than the other models and also in addition to predicting power, this model has a high speed in terms of training and testing speed than other models. Chitsaz and Banihabib [6] in addition to using different multi-criteria decision-making methods, used three Mean, Borda and Copland techniques to prioritize flood management alternatives in the Gorganrud catchment to achieve the best alternatives. Their results showed that Non-compensatory and ELECTRE 3 models are superior to other models. Chitsaz and Azarnivand [7] using the Multi-criteria evaluation technique, performed water shortage management in arid areas of Yazd province with the help of the SWAT model and hierarchical analysis method. The results showed that facilitating private sector participation in industry and tourism could be considered as the first priority and alternative to reduce water shortages in the agricultural sector in the province. Golfam et al. [8] used VIKOR multi-criteria optimization and compromise solution and fuzzy order weighted average (FOWA) methods in a study to evaluate the performance of Aydoghmush dam reservoir in East Azerbaijan province and predict climate change in order to ensure Sustainability of agricultural water supply. The results showed that multi-criteria decision-making methods offer the best alternatives for managing water supply with climate change. Khoshand et al. [9], established a model based on the AHP method for the assessment of different alternatives for energy recovery from the waste in Tehran. The results indicated the best suitable alternative is anaerobic digestion due to better environmental and economic aspects comparing to the other options. Moreover, the results of sensitivity analyses showed anaerobic digestion is the most stable alternative in comparison to the other alternatives. Ashrafi and mahmoudi [10], tried to model the water resources system of Great Karun watershed using the water evaluation and planning system (WEAP) model as a semi-distributed system in the southwest of Iran. They applied the Harmony Search (HS) Optimization Algorithm to calibrate the simulation model. Results revealed that regardless of the quality parameters of the flow, urban, industrial, agricultural and aquaculture demands at the basin level have been
satisfactorily fulfilled in the study period, as well as the comparison of the achieved results with the observed data indicated the accuracy of the calibrated model. Moradi et al. [11] used statistical tests such as T-test and Kruskall-Wallis tests to study and analyze the difference between the quantitative parameters before and after constructing dams and the effect of different factors on water quality. Their results showed that the values of the investigated water quality parameters (except EC value) before constructing dams were significantly different from the values after constructing dams in reservoir downstream stations. They concluded that the constructed reservoir dams affected the water quality characteristics of Marun and Roudzard rivers in the studied basin.

In the present study, multi-criteria decision-making models were used to evaluate the performance of the Golest Dam reservoir in Golestan province in Iran according to its evaluation indicators in optimal conditions. Moreover, the efficiency of these models was compared. Volumetric and time-based reliability, reversibility, vulnerability, and optimization objective function was used to evaluate the performance of Differential Evolution, whale, Gray Wolf, and Crow Search optimization algorithms as decision alternatives. Finally, the Mean, Borda, and Copland techniques were used to select the most appropriate multi-criteria decision-making models in order to solve the problem of optimization of the single reservoir system of Golestan Dam.

2. Materials and methods

2.1. The study area and statistical information

Golest Dam is constructed on the Gorganrud River, 12 km northeast of Gonbad. The purpose of its construction was to meet the needs of agriculture, improve and develop the lands on the right bank of the Gorganrud River, meet the water needs of the industrial sector, environmental demands, and increasing the life of Voshmguir dam and flood controlling. The volume of this reservoir at normal level (100 meters above sea level) is 48 million cubic meters and at overflow level is 86 million cubic meters. Figure 1 shows the geographical location of the study area.
2.2. Reservoir optimal operation model

The planning includes a period of 150 months from March 2005 to July 2016. The model input information consists of the monthly time series of river flow volume, evaporation volume from the reservoir surface, and the volume of downstream needs of the dam. The reservoir release volume is defined as the optimization decision variable. The objective function is to minimize the sum of the relative squares in water supply to maximum demand per month. Therefore, the objective function and related constraints can be considered according to the following relations [12]:

Minimize \( F_{\text{Release}} = \frac{1}{n} \sum_{t=1}^{n} \left( \frac{D_t - R_t}{D_{\text{max}}} \right)^2 \)  \( (1) \)

\( S_{(t+1)} = S_t + Q_t - R_t - S_p_t - (ev_t \times Av_t/1000) \)  \( (2) \)

\( S_{\text{min}} \leq S_t \leq S_{\text{max}} \)  \( (3) \)

\( R_{\text{min}} \leq R_t \leq R_{\text{max}} \)  \( (4) \)

\( 0 \leq R_t \leq D_t \)  \( (5) \)

\( S_{p_t} = \begin{cases} S_t + Q_t - \left( ev_t \times \frac{Av_t}{1000} \right) - S_{\text{max}} & \text{if } S_t + Q_t - (ev_t \times Av_t/1000) \geq S_{\text{max}} \\ 0 & \text{if } S_t + Q_t - R_t \leq S_{\text{max}} \end{cases} \)  \( (6) \)

Where, \( F_{\text{Release}} \) is the water release objective function, \( D_t \) is the total amount of water demand in the \( t^{\text{th}} \) month, \( R_t \) is the amount of release from the reservoir in the \( t^{\text{th}} \) month, \( D_{\text{max}} \) is the maximum volume of water demand, \( S_t \) and \( S_{(t+1)} \) Reservoir storage volume at the beginning and end of \( t^{\text{th}} \) month, \( Q_t \) is the volume of inflow to dam reservoir in \( t^{\text{th}} \) month, \( ev_t \) Evaporation...
height from dam reservoir surface in \( t \)th month, \( S_{\text{max}}, S_{\text{min}} \) are maximum and minimum volume
Dam reservoir in \( t \)th month, respectively. \( S_{\text{p}} \) is volume of reservoir overflow in \( t \)th month, \( n \) is the
planning period, \( A\bar{v}_t \) is average reservoir surface in \( t \)th month and in \( A\bar{v}_t = aS_t^2 + bS_t + c \), \( a, b \text{ and } c \) are a constant coefficients of the surface-volume equation of the reservoir [10]. The
following penalty functions are used for \( \forall t = 1, 2, \ldots T \) to ensure the satisfaction of constraints
3, 4, and 5[12].

\[
P F_1 = A' \left( \frac{|S_{\text{min}} - S_t|}{S_{\text{max}} - S_{\text{min}}} \right)^2 + B' 
\]

\[
P F_2 = C' \left( \frac{R_t - D_{\text{max}}}{D_{\text{Max}}} \right)^2 + D' 
\]

\[
P F_3 = E' \left( \frac{R_{t+1} - D_{\text{max}}}{D_{\text{Max}}} \right) + F' 
\]

Where, \( PF_1, PF_2 \) and \( PF_3 \) are the penalty functions and the coefficients \( A' \) to \( F' \) are the
positive constants of the penalty function.

### 2.3. Reservoir efficiency indices

In this study, indices of reliability (volumetric and time-based), vulnerability, and
reversibility were used. If the purpose of the reservoir is to meet the water demand, the reliability
index is defined as the probability of meeting the volume of demand (volumetric reliability) or a
certain percentage of demand in a certain period of time (time-based reliability) according to the
following relationship [12].

\[
\alpha = P\{X_t \in Sa\} 
\]

(10)

Where \( \alpha \) is the reliability index, \( X_t \) is the status of the system in \( t \)th time period, \( Sa \) is the
optimal state of the system and \( p \) is the probability of supply in \( t \)th time period.

Vulnerability index can be defined as the size of system failures to the total volume of water
demand according to the following relationship [12]:

\[
u = \sum_{j \in Fa} \frac{(D_t - R_t)}{V_t} 
\]

(11)

Where, \( u \) is the Vulnerability index, \( D_t - R_t \) is the volume of shortage in time \( t \) and \( V_t \) is the
total volume of demand in \( t \)th time period.

The reversibility index indicates the probability of the system returning to the desired state
(\( Sa \)) after failure (\( Fa \)), which is defined by the following relation [12].

\[
\beta = P\{X_{t+1} \in Sa | X_t \in Fa\} 
\]

(12)

Where, \( \beta \) is the index of reversibility and \( Fa \) is the failure state of the system [12].
2.4. Differential Evolution algorithm

The Differential Evolution (DE) algorithm is a simple and powerful algorithm for solving optimization problems. The DE algorithm contains two important parameters, the mutation operator \( SF \) and the other is the \( (CR) \) crossover probability, which is as follows [13].

2.4.1. Mutation operator

The mutation operator is based on the following relation in the Differential Evolution algorithm [13].

\[
m_i^{(g+1)} = X_{r1}^g + SF \times (X_{r2}^g - X_{r3}^g)
\]  

(13)

Where \( m_i^{(g+1)} \) is the mutating factor in \( g + 1 \)th generation and \( X_{r1}^g, X_{r3}^g \) and \( X_{r2}^g \) indicate other factors in the population. In addition, \( SF \) is a constant value that indicates the cause of the mutation.

2.4.2. Crossover operator

In the process of crossover, the algorithm randomly selects an individual from the population to evolve into population diversity so that the individual can be either an ordinary person \( X_i^{(g)} \) or a mutant \( m_i^{(g+1)} \) [13].

2.4.3. Selection operator

At this stage, the measurement vector obtained from the previous stage and the target member selected in the first stage are evaluated according to the objective function, and if the measurement vector is more valuable than the target member, it is considered as a member of the next generation. Otherwise, the target member will be considered one of the next generation population. The following equation indicates selecting between the measurement vector and the target member [13].

\[
X_i^{(g+1)} = \begin{cases} 
c_i^{(g+1)} & \text{if } f(c_i^{(g+1)}) \leq f(c_i^{(g)}) \\
X_i^{(g)} & \text{otherwise} 
\end{cases}
\]

(15)

Where, \( f \) represents the value of the fitness function of the problem.

2.5. Whale optimization algorithm (WOA)

In this algorithm, while depicting the social behavior of humpback whales using three operators of prey siege, bubble-net attack method (exploitation stage), and prey search (exploration stage), the search agents’ position is updated in each iteration. In this algorithm, optimization for the best search agent is performed based on equations 16 and 17[14].
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\[ |\vec{C} \cdot \vec{X}^*(t) - X(t)| \] \hspace{1cm} (16)

\[ (t + 1) = \vec{X}^*(t) - \vec{A} \cdot \vec{D} \] \hspace{1cm} (17)

\[ \vec{A} = 2\vec{a} \cdot \vec{r} - \vec{a} \] \hspace{1cm} (18)

\[ \vec{C} = 2 \cdot \vec{r} \] \hspace{1cm} (19)

Where, \( t \) is the current iteration, \( \vec{A} \) and \( \vec{C} \) are the coefficient vectors, \( \vec{X}^* \) is the location vector of the best solution obtained at present, \( \vec{X} \) is the location vector [11]. \( \vec{a} \) is reduced linearly from two to zero over iterations and \( \vec{r} \) is a random vector between zero to one [11]. In the bubble net attack method, humpback whales swim around the prey along a contractile circle and simultaneously in a spiral path (Figure 2). The aggressive behavior of the bubble net attack can be demonstrated mathematically with the following equations [14]:

\[ \vec{D} = |\vec{C} \cdot \vec{X}^*(t) - X(t)| \] \hspace{1cm} (20)

\[ \vec{X}(t+1) = \vec{D} \cdot e^{br} \cdot \cos(2\pi r) + \vec{X}^*(t) \] \hspace{1cm} (21)

Where, \( \vec{D} \) refers to the distance \( i^{th} \) from the whale to the prey (the best solution ever obtained), \( b \) is a constant for defining a logarithmic spiral shape, \( r \) is a random number between -1 and +1.

To model this behavior, it is assumed that the whale selects one of the mechanisms of contraction siege or spiral model with a 50% probability to improve the positions of the whales during the optimization process. Its mathematical model is defined as the following equation [14]:

\[ \vec{X}(t+1) = \begin{cases} \vec{X}^*(t) - \vec{A} \cdot \vec{D} & \text{if } p < 0.5 \\ \vec{D} \cdot e^{br} \cdot \cos(2\pi r) + \vec{X}^*(t) & \text{if } p \geq 0.5 \end{cases} \] \hspace{1cm} (22)

Where \( P \) is a random number between zero and one. In order to increase the capability of the
exploration phase, a large search strategy has been used in the whale algorithm. Search agents update their position according to a random factor in the population. Since $\vec{A}$ with random values between -1 to +1 indicates the proximity of the search agent to the reference whale, the search for prey behavior can be mathematically as follows [14]:

\[
\vec{D} = |C \cdot \vec{X}_{\text{rand}} - \vec{X}(t)| \\
\vec{X}(t+1) = \vec{X}_{\text{rand}} - \vec{A} \cdot \vec{D}
\] (23) (24)

In which, $\vec{X}_{\text{rand}}$ is a random position vector selected from the current population.

2.6. Gray Wolf Optimization algorithm (GWO)

The Gray Wolf Optimization Algorithm (GWO) is inspired by the structure of wolves’ social behavior during hunting, and all members of the group have a very precise hierarchy of social dominance that includes four main ranks and is modeled in a pyramid structure like Figure 3 with reduced dominance from top to bottom. These four groups include leader or alpha wolves that manage hunting, beta wolves that assist the alpha group in the decision-making process, delta wolves that include puppy wolves, and omega wolves that have the least rights over other members of the herd and play the role of victim in the herd [15].

\begin{center}
\textbf{Figure 3. The social hierarchy of Gray Wolves [16].}
\end{center}

2.6.1. prey siege modeling process of Gray Wolves

The mathematical model of the siege behavior is presented in the following equations in which, $t$ is the number of iterations, $A$ and $C$ are coefficient vectors, $\vec{X}_{\text{prey}}$ is the location vector of prey and $\vec{X}_{\text{G Wolf}}$ is the location vector of each gray wolf [15].

\[
\vec{D} = |C \cdot \vec{X}_{\text{prey}}(t) - \vec{X}_{\text{G Wolf}}(t)| \\
\vec{X}_{\text{G Wolf}}(t+1) = \vec{X}_{\text{prey}}(t) - \vec{A} \cdot \vec{D}
\] (25) (26)

The vectors $A$ and $C$ are also calculated as follows [15]:

\[
\vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a}
\] (27)
\[
\bar{C} = 2, \bar{r}_2 \\
a = 2 - \text{iter} \times \left(\frac{2}{\text{Max} - \text{iter}}\right)
\]

Where the components of \(\bar{a}\) are reduced linearly from 2 to zero during successive iterations. In addition, \(\bar{r}_1\) and \(\bar{r}_2\) are random vectors between zero and one [15].

### 2.6.2. Hunting procedure

In this process, it is assumed that alpha, beta, and delta wolves have a better knowledge of the potential position of the prey. Other search agents, including omega, are required to update their location based on the position of the best search agents (equations 30 to 32) [15].

\[
\begin{align*}
\bar{D}_a &= [\bar{C}_1.X_a - \bar{X}], \bar{D}_\beta = [\bar{C}_2.X_\beta - \bar{X}], \bar{D}_\delta = [\bar{C}_3.X_\delta - \bar{X}] \\
\bar{X}_1 &= \bar{X}_a - \bar{A}_1 \cdot (\bar{D}_a), \bar{X}_2 &= \bar{X}_\beta - \bar{A}_2 \cdot (\bar{D}_\beta), \bar{X}_3 &= \bar{X}_\delta - \bar{A}_3 \cdot (\bar{D}_\delta) \\
\bar{X}(t + 1) &= \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3}{3}
\end{align*}
\]

When the prey is surrounded by wolves and stopped moving, the attack led by Alpha Wolf begins. The modeling of this process is done using the reduction of \(\bar{a}\). Since \(\bar{A}\) in equation 27 is a random vector in the interval between \([2a, 2a]\), in case \(|A| < 1\), the alpha wolf will approach the prey and in case \(|A| > 1\) the wolf will stay away from the prey. In this algorithm, all wolves must update their position according to the position of alpha, beta and delta wolves [15].

### 2.6.3 Searching procedure

The function of the searching phase is exactly the opposite of the attack process, in searching phase, the wolves move away from each other to track the prey (\(|A| > 1\)), while after tracking the prey, the wolves’ approach each other in the attack phase (\(|A| < 1\)). This strategy is referred to as the mechanism of divergence in search and convergence in attack [15].

### 2.7 Crow search optimization algorithm (CSA)

Crows are one of the most intelligent birds. While watching the other birds, crows know how to hide their food so that other birds cannot steal their food after leaving the place. In fact, they use their personal experiences to predict the behavior of other birds and are able to determine the safest route to protect their hiding places. The description of the crow search optimization algorithm (CSA) is as follows [17].

At the beginning of the algorithm, it is assumed that the algorithm space consists of \(d\) dimensions and \(n\) crows. The position of each crow at each stage of iteration is defined based on the vector \(X^{iter} = [X_1^{iter}, ..., X_d^{iter}]\). Each crow has a memory that remembers where the food was hidden. At each iteration, the crow’s hidden location is specified as \(m^{iter}\). This is the best position the crow has chosen to hide its food. In addition, the best place to hide food is stored in the crow’s memory and they continue to search in the surrounding area to find the best place to hide food in case of emergency. In other words, at this stage the \(i^{th}\) crow decides to
follow the \( j \)th crow. Suppose that at each stage, the crow also wants to see where its food is hiding (\( m_j, \text{iter} \)). At this point, the \( i \)th crow decides to chase the \( j \)th crow to access his food. Therefore, in these conditions, the following two cases may exist [14].

1. The \( j \)th crow does not know that the \( i \)th crow is chasing it, and therefore the crow accesses the location of the crow. Therefore, the new position of the \( i \)th crow is obtained based on the following relation [17].

\[
X_{i, \text{iter}+1} = X_{i, \text{iter}} + r_i \times f l_{i, \text{iter}} \times (m_{j, \text{iter}} - X_{i, \text{iter}})
\]  

(33)

Where, \( r_i \) is a random number with a uniform distribution, and \( 1 < r_j < 0 \), \( f l_{i, \text{iter}} \) is the length of the crow \( i \)th. Figure 6 shows the effect of \( f l_{i, \text{iter}} \) on an algorithm search capability [17].

The small values of this parameter lead to local search and large values lead to an absolute optimal search. Figure 6 shows that if the value of \( f l \) is less than one, the next position of crow \( i \)th is determined between \( m_{j, \text{iter}}, X_{i, \text{iter}} \) along the distance line. In addition, if the above parameter is bigger than one, then the new position of the crow may be on the distance line and after \( m_{j, \text{iter}} \) as shown in Figure 4 [17].

2. The \( j \)th crow knows that \( i \)th crow is chasing it. As a result, \( j \)th crow tries to deceive \( i \)th crow and take it to another position (equation 35) [9]. Where, \( r_j \) is a random number with a uniform distribution between zero and one and \( A P_{i, \text{iter}} \) is the awareness probability that sometimes \( j \)th crow is at any iteration. In the mentioned algorithm, the ability of variation and resonance is controlled based on the AP parameter [17].

2.7.1 CSA steps:

2.7.1.1 Decision variables and constraints are defined. The number of crows (\( N \)), flight length (\( fl \)), maximum iteration, and awareness probability (AP) are determined.

2.7.1.2 Since in the first iteration the crows have no experience, it is assumed that they have hidden their food in their original positions. Therefore, crows are randomly placed in the \( D \) dimensional search space. Each of the crows represents a possible solution to the problem and \( d \) is the number of decision variables. Therefore, the position and memory of the crows are stabilized as:

\[
\text{Crows} = \begin{bmatrix}
x_1^1 & \ldots & x_1^d \\
\vdots & \ddots & \vdots \\
x_N^1 & \ldots & x_N^d \\
\end{bmatrix} \quad \text{Momory} = \begin{bmatrix}
m_1^1 & \ldots & m_1^d \\
\vdots & \ddots & \vdots \\
m_N^1 & \ldots & m_N^d \\
\end{bmatrix}
\]  

(34)

\[
X_{i, \text{iter}+1} = \left[ X_{i, \text{iter}} + r_i \times f l_{i, \text{iter}} \times (m_{j, \text{iter}} - X_{i, \text{iter}}) \right] \left\{ \begin{array}{l}
\text{a random(position)} \leftarrow \text{otherwise} \\
\end{array} \right.
\]  

(35)
2.7.1.3 In the third step, the objective function is evaluated and its values are calculated for each crow.

2.7.1.4 Then the crows’ memory is updated according to the following equation:

\[
m_{\text{iter}+1} = \begin{cases} x_{\text{iter}+1} \leftarrow f(x_{\text{iter}}) \text{ is (better) than } f(m_{\text{iter}}) \\
m_{\text{iter}} \quad \text{otherwise}
\end{cases}
\]

(36)

2.7.1.5 At this stage, the convergence condition is controlled and if it is satisfactory, the algorithm is completed [17].

2.8 Multi-criteria decision-making methods (MCDM)

2.8.1 TOPSIS technique

In this decision-making method, there are a number of alternatives and criteria so that these alternatives should be ranked according to the criteria. In this method, the decision matrix is first formed, which includes criteria (in columns) and alternatives (in rows). Then, by dividing each value by the square of the sum of the squares of that column of the decision matrix column, it becomes a dimensionless matrix. Then, by multiplying the weight of the obtained criteria by other methods such as AHP, entropy, etc. in the normal matrix, the weighted matrix is obtained. Because the criteria are either positive or negative. The ideal and anti-ideal solutions should be obtained by calculating the distance from the ideal and anti-ideal solutions based on relations 37 and 38 [18].

\[
d_i^+ = \sqrt{\sum_{j=1}^{n} (V_{ij} - A_j^+)^2}
\]

(37)

\[
d_i^- = \sqrt{\sum_{j=1}^{n} (V_{ij} - A_j^-)^2}
\]

(38)

Afterward, the similarity and ranking index of the alternatives is calculated using Equation 39 so that the closer the index to the number one, the superior the alternative will be [18].
\[ cl_i^* = \frac{d_i^-}{d_i^- + d_i^+} \]  

\[ f_{ij} = X_{ij} / \sqrt{\sum_{i=1}^{m} X_{ij}^2} \]  

\[ S_i = \sum_{j=1}^{n} W_j \left( f_j^* - f_{ij} \right) / f_j^* - \bar{f}_j \]  

\[ R_i = \max \left\{ W_j \left( f_j^* - f_{ij} \right) / f_j^* - \bar{f}_j \right\} \]  

\[ Q_i = V \left[ \frac{S_i - \bar{S}}{S^* - \bar{S}} \right] + (1 - V) \left[ \frac{R_i - \bar{R}}{R^* - \bar{R}} \right] \]  

2.8.2 VIKOR model

This model is based on the adaptive planning of multi-criteria decision-making issues. In this model, the decision matrix dimensionless is as follows [19]:

\[ f_{ij} = X_{ij} / \sqrt{\sum_{i=1}^{m} X_{ij}^2} \]  

The calculation of the amount of benefit (S) and the amount of regret (R) is as follows [19]:

\[ S_i = \sum_{j=1}^{n} W_j \left( f_j^* - f_{ij} \right) / f_j^* - \bar{f}_j \]  

\[ R_i = \max \left\{ W_j \left( f_j^* - f_{ij} \right) / f_j^* - \bar{f}_j \right\} \]  

Where \( W_j \) is the desired weight for the criterion \( j \) and for the positive criteria \( f_j^* = \max_i f_{ij} \) and \( \bar{f}_j = \min_i f_{ij} \) as well as for the negative criteria \( f_j^- = \min_i f_{ij} \) and \( \bar{f}_j = \max_i f_{ij} \). The fraction \( f_j^*-f_{ij} / f_j^- - \bar{f}_j \) is equal to the distance rate from the ideal state for the criteria \( j=1, 2, \ldots, n \) in alternative \( i \), which is averaged over the usefulness of these distances, but the maximum distance from the ideal state of the criteria is calculated for each alternative only at the regret rate. The value of Q (VIKOR index) is calculated as follows [19]:

\[ Q_i = V \left[ \frac{S_i - \bar{S}}{S^* - \bar{S}} \right] + (1 - V) \left[ \frac{R_i - \bar{R}}{R^* - \bar{R}} \right] \]  

Where \( \bar{S} = \min S_i \), \( \bar{R} = \min R_i \), \( R^* = \max R_i \) and \( S^* = \max S_i \), so that \( \frac{S_i - \bar{S}}{S^* - \bar{S}} \) is the average distance rate from the ideal solution for alternative \( i \) and \( \frac{R_i - \bar{R}}{R^* - \bar{R}} \) indicates the maximum distance from the ideal solution for alternative \( i \). The VIKOR parameter \( (v) \) is selected based on the degree of agreement of the decision-making group. If its value is more than 0.5, the importance of the average rate of distance from the ideal state of the alternatives increases, and if its value is less than 0.5, the importance of the maximum distance rate from the ideal state of the alternatives increases. Then, the alternatives are sorted from small to large based on the values of R, S and Q, and the final ranking of the model is based on the values of Q, according to the Q values, an alternative is selected as the best alternative that can meet the following conditions [19].
2.8.2.1 If the alternatives A1 and A2 represent the first and second top, the following equation is established:

\[ Q(A_2) - Q(A_1) \geq \frac{1}{n - 1} \]  

(44)

2.8.2.2 Alternative A1 should be recognized as the top rank in one of the groups R and S at least. When the first condition is not met, a set of alternatives such as A1, A2…, Am are selected as the top alternatives. As the maximum value of m is calculated according to the following equation [19].

\[ Q(A_m) - Q(A_1) < \frac{1}{n - 1} \]  

(45)

When the second condition is not met, two alternatives A1 and A2 are selected as the best alternatives [19].

2.8.3 Simple Additive model (SAW)

In this model, the score of alternatives is calculated from the following equation [20].

\[ A^* = \{ A_1 | \max_i \sum_j = 1^m W_j r_{ij} \} \]  

(46)

Where \( r_{ij} = X_{ij}/\max_i \{X_{ij}\} \) are the elements of the dimensionless matrix, \( X_{ij} \) is the function of alternative i on the criterion j and \( W_j \) is the weight of the criterion j [20].

2.8.4 ELECTRE type I method

This method, like other decision models, is used to select the superior alternative between several alternatives. The operation of this method is similar to TOPSIS and seeks to prioritize alternatives through different criteria. In this method, the weight of the criteria must be predetermined through other methods. In the ELECTRE I method, after creating a decision matrix consisting of alternatives and criteria, normalizing the matrix using the following equation should be noticed; because quantitative criteria have their own measurement scale, which makes it impossible to compare them with each other [18].

\[ n_{ij} = a_{ij}/\sqrt{\sum a_{ij}^2} \]  

(47)

\( n_{ij} \) is the normal value of alternative i of the criterion j. Then the dimensionless matrix must be weighted using Shannon entropy (equations 48-52). M is the number of alternatives [18].

\[ r_{ij} = \frac{X_{ij}}{\sum_{i=1}^m X_{ij}} \]  

(48)
\[ k = \frac{1}{\ln (m)} \]  
\[ E_j = -k \sum_{i=1}^{m} (r_{ij} \cdot \ln r_{ij}) \]  
\[ d_j = 1 - E_j \]  
\[ w = d_j / \sum_{j=1}^{n} d_j \]  

\( E_j \) is the entropy value and \( d_j \) is the degree of deviation of the \( j \) index. In the next step, the alternatives are compared in pairs and the criteria in which the \( i^{th} \) alternative is superior to the \( j^{th} \) are placed in the coordination set and the rest in the non-coordination set [18].

\[ c_{ij} = \sum w_j \]  

Then, the discordance matrix is formed based on the following equation [15].

\[ d_j = \max |V_j - V_j| / \max |V_i - V_j| \]  

In the next step, the Boolean matrix of coordination is formed. This matrix of alternatives whose utility is below the threshold becomes zero and the rest becomes one. The threshold is obtained from the following relation [18].

\[ \tilde{c} = \frac{\sum_c c \geq \tilde{c}}{m \times (m-1)} c \geq \tilde{c} \quad B = 1 \]  
\[ \tilde{d} = \frac{\sum_d d \geq \tilde{d}}{m \times (m-1)} d \geq \tilde{d} \quad h = 0 \]  

Then, the final dominance matrix \( (Z) \) is obtained by multiplying the Boolean Matrix \( B \) by the Boolean Matrix \( H \) based on the following equation, which indicates the relative preference of the alternatives [18].

\[ Z = H \times B \]  

**2.8.5 LINMAP method**

Linear Programming Method for Multidimensional Preference Analysis (LINMAP) is one of the latest well-known multi-attribute decision-making (MADM) methods that seek to find the alternative that has the shortest distance with the ideal one. In this method, by comparing the Euclidean distance of the alternatives with the best alternative, the most suitable alternative is selected as following:

2.8.5.1 The decision maker identifies the elements of the decision matrix (if necessary, the decision matrix is normalized by the linear method).
2.8.5.2 The decision maker prioritizes the relation of the alternatives using the set \( \Omega = \{ (K,L) | X_K \geq X_L. \ (K,L = 1,2, ..., m) \} \).

The omega complex normally has \( (m-1)/2 \) (m is the number of alternatives) [21].

2.8.5.3 The decision maker determines the value of \( X_{ij} \) of the alternatives (\( A_i \)) according to the criteria (\( C_j \)).

2.8.5.4 In this step, the decision matrix is formed.

2.8.5.5 The linear programming model is made using the following equation [21].

\[
\begin{align*}
\min \ & \sum_{(k,l)\in\Omega} Z_{kl} \\ S.t. \ & \sum_{j=1}^{n} W_j \sum_{(k,l)\in\Omega} (x_{Lij}^2 - x_{Kij}^2) - 2 \sum_{j=1}^{n} V_j \sum_{(k,l)\in\Omega} (x_{Lij} - x_{Kij}) = h \\
& \sum_{j=1}^{n} W_j(x_{Lij}^2 - x_{Kij}^2) - 2 \sum_{j=1}^{n} V_j(x_{Lij} - x_{Kij}) + Z_{kl} \geq 0 \ (k,l \in \Omega) \\
& \sum_{j=1}^{n} W_j = 1 \ Z_{kl} \geq 0 \ (k,l \in \Omega)
\end{align*}
\]

\( h \) is an arbitrary constant.

2.8.5.6 In this step, the linear programming model is solved using the simplex method.

2.8.5.7 The weight of each of the criteria \( W_j \) and the vector \( r_j^* \ (j = 1,2, ..., m) \) is an ideal representation of the \( j \)th index, which is obtained using the following equation [21].

\[
V_j = W_j r_j^* \tag{59}
\]

\( W_j \) also indicate the degree of importance of each indicator.

2.8.5.8 The value of \( S_i (i = 1,2, ..., m) \) is calculated using the following equation for each alternative [21].

\[
S_i \sum_{j=1}^{n} W_j(x_{ij} - r_j^*)^2 \tag{60}
\]

2.8.5.9 The alternatives are ranked based on the ascending values of \( S_i (i = 1,2, ..., m) \).

2-8-6- Combinative Distance-based Assessment

The Combinative Distance-based Assessment Multi-Criteria Decision Making Model
(CODAS), first proposed by Keshavarz Ghorabaee et al [22]. This model is one of the multi-criteria decision making methods for selecting the best alternative, which is based on the ranking of alternatives according to the number of criteria. In this method, first the Euclidean distance and then the Taxicab distance is calculated based on the difference with the negative ideal point. Any alternative that has the greatest distance from the negative ideal is the best alternative in the CODAS method. In general, assuming m criteria and n alternatives, the steps of this method can be described as follows:

The first step is to create a decision matrix such as equation below [22]:

\[ X = [x_{ij}]_{n \times m} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \] (61)

The second step is to normalize the decision matrix. Normalization is done using the following equations. If the criterion has a positive aspect (profit), the first relation and if it has a negative aspect (cost), the second relation is used [22].

\[ n_{ij} = \begin{cases} \frac{x_{ij}}{\max_{l} x_{ij}} & \text{if } j \in N_b \text{ for Positive Criteria} \\ \frac{\min_{l} x_{ij}}{x_{ij}} & \text{if } j \in N_c \text{ for Negative Criteria} \end{cases} \] (62)

The third step is to create a normal weighted matrix. That is, the weight of the criteria must be multiplied by the normal matrix. This weight can be obtained from other methods, including the Shannon entropy method, the Best-Worst method (BWM), or the AHP method.

Then we have to calculate the Euclidean and Taxicab distances from the negative ideal. These distances are obtained from the following equations. In these equations, nsj is the negative ideal of criteria [22].

\[ E_i = \sqrt{\sum_{j=1}^{m} (r_{ij} - ns_j)^2} \] (63)

\[ T_i = \sum_{j=1}^{m} |r_{ij} - ns_j| \] (64)

In the fifth step, the relative evaluation matrix is obtained using the following equation. Where, Ψ represents a threshold function to determine the equality of the Euclidean distance of two alternatives [19].

\[ h_{ik} = (E_i - E_k) + (\psi(E_i - E_k) \times (T_i - T_k)) \] (65)

Finally, by summing the h_{ik} values of the alternatives as the parameter named H_i, they can be ranked. The larger the H_i value of the alternatives, the better the ranking alternative [21].
In a multi-criteria decision problem, several multi-criteria decision methods may be used, the results of which are not always the same. In fact, in such cases, the question that arises is which alternatives should be chosen. To make decisions on very important problems, decision makers do not limit themselves to one method and use integration methods to overcome this situation [23]. These methods include Mean, Borda, and Copeland.

2.9 Integration multi-criteria decision-making methods

2.9.1 The Mean method

This method is known as the mean of the rankings. In this method, the arithmetic mean of the obtained rankings is determined and the alternatives are prioritized accordingly. It is obvious that alternatives with higher arithmetic mean will be preferred [23].

2.9.2 The Borda method

In this method of decision making, a pairwise comparison matrix is created between the alternatives. If according to the various multi-criteria decision-making methods, the number of alternatives preferred over another alternative is greater than the number of defeats of that alternative over another alternative, the number 1 is placed in the pairwise comparison matrix and zero if it is vice versa. The number 1 means that the row takes precedence over the column, and the number zero means that the column takes precedence over the row. After examining the alternatives, a pairwise comparison matrix is formed and the sum of the elements of each row shows the number of the dominance of each alternative, and the alternatives are prioritized based on the number of dominances [23].

2.9.3 The Copeland method

This method starts with the end of the Borda method. The Copeland method calculates not only the number of dominance but also the number of defeats for each alternative. The score that Copeland gives to each alternative is calculated by subtracting the number of defeats from the number of dominances. This is the modified method of Borda, with the difference that in addition to the number of dominance (total elements of each row), the number of defeats (total elements of each column) is also used in prioritization. For this purpose, the alternatives are prioritized based on the difference between the values of the number of dominance and the number of defeats [23].

3. Results and Discussion

In this study, to evaluate the results of optimization with the above algorithms, the algorithms were developed in MATLAB R2015 software, the results of which are presented in Table 1 with 10 running steps and 100 iterations in each running step. It is noteworthy that these values are compared with each other and in comparison, with the absolute optimal response (19.41) obtained from the nonlinear programming method by GAMS software. Therefore, the average response obtained from the GWO is 1.08 times the absolute optimal response and the average response of the WOA, DE, and CSA are 1.49, 1.29, and 1.19 times the absolute optimal response, respectively. In addition, the coefficients of variation of the GWO are 2, 113, and 1.43 times smaller than the WOA, DE, and CSA, respectively. These findings indicate the better performance of the responses obtained from the GWO in achieving relative optimal values.
According to Figure 5, since the minimum values obtained from the optimization of the GWO are much more appropriate than the values obtained from the optimization with other algorithms, the performance of the GWO can be clearly seen in achieving the lower values. Table 2 shows the different performance evaluation indicators of the algorithms, along with the weights obtained from the Shannon entropy method for each of the evaluation criteria and each of the algorithms. As can be seen, in all model evaluation parameters including reliability, reversibility, vulnerability, and objective function, the GWO performs better than other algorithms [24].

**Figure 5. Convergence of responses in evolution algorithms**

**Table 1. Comparison of different evolution algorithms in the Golestan Dam operation**

<table>
<thead>
<tr>
<th>Running step in MATLAB</th>
<th>Whale algorithm</th>
<th>Crow Search Algorithm</th>
<th>Gray wolf algorithm</th>
<th>Differential Evolution algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29.31</td>
<td>22.90</td>
<td>21.20</td>
<td>26.79</td>
</tr>
<tr>
<td>2</td>
<td>28.88</td>
<td>23.10</td>
<td>20.90</td>
<td>23.83</td>
</tr>
<tr>
<td>3</td>
<td>29.78</td>
<td>23.11</td>
<td>21.34</td>
<td>25.17</td>
</tr>
<tr>
<td>4</td>
<td>28.26</td>
<td>23.09</td>
<td>21.16</td>
<td>24.35</td>
</tr>
<tr>
<td>5</td>
<td>29.10</td>
<td>23.08</td>
<td>21.10</td>
<td>25.55</td>
</tr>
<tr>
<td>6</td>
<td>29.23</td>
<td>23.12</td>
<td>21.20</td>
<td>25.63</td>
</tr>
<tr>
<td>7</td>
<td>28.90</td>
<td>23.09</td>
<td>21.90</td>
<td>24.95</td>
</tr>
<tr>
<td>8</td>
<td>28.92</td>
<td>23.08</td>
<td>21.15</td>
<td>24.79</td>
</tr>
<tr>
<td>9</td>
<td>28.86</td>
<td>23.10</td>
<td>21.00</td>
<td>25.05</td>
</tr>
<tr>
<td>10</td>
<td>28.75</td>
<td>23.11</td>
<td>21.98</td>
<td>24.95</td>
</tr>
<tr>
<td>Mean</td>
<td>29.00</td>
<td>23.15</td>
<td>21.09</td>
<td>25.11</td>
</tr>
<tr>
<td>The worst answer</td>
<td>29.78</td>
<td>23.80</td>
<td>21.34</td>
<td>26.79</td>
</tr>
<tr>
<td>The best answer</td>
<td>28.26</td>
<td>22.90</td>
<td>21.90</td>
<td>23.83</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.014</td>
<td>0.010</td>
<td>0.007</td>
<td>0.79</td>
</tr>
<tr>
<td>Absolute optimal answer</td>
<td>19.41</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2. Indicators obtained from evolution algorithms (decision matrix in percentage)

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Volumetric reliability</th>
<th>Time-based reliability</th>
<th>Reversibility</th>
<th>Vulnerability</th>
<th>The objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>(WOA)</td>
<td>0.73</td>
<td>0.8</td>
<td>0.7</td>
<td>0.2</td>
<td>29</td>
</tr>
<tr>
<td>(GWO)</td>
<td>0.94</td>
<td>0.91</td>
<td>0.95</td>
<td>0.09</td>
<td>21.09</td>
</tr>
<tr>
<td>(CSA)</td>
<td>0.87</td>
<td>0.83</td>
<td>0.76</td>
<td>0.17</td>
<td>23.15</td>
</tr>
<tr>
<td>(DE)</td>
<td>0.7</td>
<td>0.67</td>
<td>0.56</td>
<td>0.23</td>
<td>25.11</td>
</tr>
</tbody>
</table>

In this study, decision alternatives were ranked based on performance assessment criteria using six multi-criteria decision-making methods: TOPSIS, VIKOR, LINMAP, CODAS, ELECTRE Type I, and Simple Additive Weighting (SAW) (Table 3). Then, considering the non-uniformity of the results obtained from the above-mentioned models, integration methods of Mean, Broda, and Copeland were used to evaluate the performance of multi-criteria decision-making models and to select the appropriate models in the Golestan Dam reservoir optimization problem. As can be seen in Tables 3 and 4. According to the results in table 3, although, all integration methods have reached the same results, the performance of multi-criteria decision-making methods of TOPSIS, VIKOR, and LINMAP can be considered appropriate in solving the problem of Golestan Dam reservoir optimization. This study shows that although the CODAS method is a very new method, it is not appropriate to use in the optimization problem. Therefore, it cannot be expected that all new multi-criteria decision-making methods are necessarily better than conventional methods. Another point is that all methods except ELECTRE Type I worked the same in ranking the first and second alternatives. It is important to note that if one method is used to make multi-criteria decision alternatives without studying other methods, it may lead to error in decision making.

Table 3. Evaluation results of multi-criteria decision models on decision alternatives (evolutionary algorithms)

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>TOPSIS</th>
<th>SAW</th>
<th>ELECTRE I</th>
<th>VIKOR</th>
<th>LINMAP</th>
<th>CODAS</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>(WOA)</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>(GWO)</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1.333</td>
</tr>
<tr>
<td>(CSA)</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2.333</td>
</tr>
<tr>
<td>(DE)</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3.333</td>
</tr>
</tbody>
</table>

Table 4. Results of integration methods to select the most appropriate decision model

<table>
<thead>
<tr>
<th>Ranking</th>
<th>Mean method</th>
<th>Borda method</th>
<th>Copeland method</th>
</tr>
</thead>
<tbody>
<tr>
<td>(WOA)</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>(GWO)</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(CSA)</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>(DE)</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>
According to Figure 6, the performance of the GWO is much better than other algorithms, so that the water release values according to the GWO need to be more consistent with the demand values. Not only this reduces the unreasonable increase in the reservoir release in the months when the water demand of the downstream lands is minimum; but also, it makes an increase in demand in the months when the water demand of the downstream lands is maximum. Figure 7 shows the greater compliance of the results obtained from the GWO regarding the storage of the reservoir with the nonlinear programming method obtained from GAMS software. This will increase the reservoir release and meet the needs of the downstream lands.

4. Conclusion

In this study, the performance of the algorithms of whale optimization (WOA), Differential Evolution optimization (DE), crow search optimization(CSA), and Gray Wolf optimization(GWO) were evaluated in order to operate the Golestan Dam reservoir with the
objective function of meeting downstream water demand. In addition, after defining the objective function and its constraints, the continuity equation, overflow, storage and release volume from the reservoir were applied to it. Then, the performance of the mentioned algorithms was compared with each other and with the absolute optimal values obtained from GAMS nonlinear programming method (19.41). The results showed that the average responses of the GWO, WOA, DE, and CSA were 1.08, 1.49, 1.29 and 1.19 times the absolute optimal response and the coefficient of variation of the answers obtained by the GWO was 2, 113 and 1.43 times smaller than the WOA, DE, and CSA, respectively. In this study, after obtaining the performance evaluation indices of each algorithm (Reliability, reversibility, and vulnerability), the objective functions, obtained by the optimization of each algorithm, were ranked using six multi-criteria decision-making methods of TOPSIS, VIKOR, LINMAP, CODAS, ELECTRE Type I and Simple Additive Weighting (SAW). Finally, integration methods (Mean, Borda, and Copeland techniques) were used to evaluate the performance of decision-making models. The results of all integration techniques indicated the superiority of the GWO. The CSA, DE, and WOA were ranked second to fourth, respectively. The use of these methods in solving the problem of Golestan Dam reservoir optimization was considered appropriate due to the similarity of the results obtained from the integration techniques with the results of TOPSIS, VIKOR and LINMAP methods. On the other hand, this study showed that the novelty of multi-criteria decision-making methods does not mean that those methods are suitable for solving different problems and are not necessarily better than conventional methods.

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**References**


Using Composite Ranking Usage to Select Appropri…


