Theoretical Criterion for Stability of Free Hydraulic Jump on Adverse Stilling Basins

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Abstract

Hydraulic jump on adverse stilling basin is an unstable phenomenon which causes some complexities in controlling of the jump. This paper considers the stability of free hydraulic jump on adverse stilling basins from a theoretical point of view. A minimum value of upstream Froude number is needed to produce the free hydraulic jump on adverse stilling basins which was presented by a theoretical equation. Also, the effects of an end sill and divergence in section on the minimum Froude number were investigated. Moreover, it is required that the bed has a minimum friction for making the jump stable which is obtained as a function of upstream Froude number and bed slope. This condition was compared with the criterion proposed by previous researchers and showed the considerable deviations at larger bed slopes. The result showed that it is impossible to establish the hydraulic jump on concrete adverse stilling basins without any appurtenances. However, diverging in section can improve the stability of hydraulic jump on adverse stilling basins.

Keywords: Hydraulic Jump, Adverse Basin, Stability, End Sill, Diverging Stilling Basin.

1. Introduction

Stilling basin is a common tool being used as an economic device to dissipate excess energy at the downstream of hydraulic structures. It is understood that any alterations in channel geometry, such as adverse slope of stilling basin, would change the characteristics of hydraulic jump and therefore its influences on downstream of such structures. It could be noted that the effect of water weight in adverse basins, as shown in Fig. a. 1, would alter the characteristics of hydraulic jump compared with the classical hydraulic jump. Hydraulic jump on an adverse slope is an interesting phenomenon which affects the sequent depths, energy loss and length of stilling basin. This phenomenon can shorten the stilling basin and the height of the side walls; hence optimize the costs of construction.

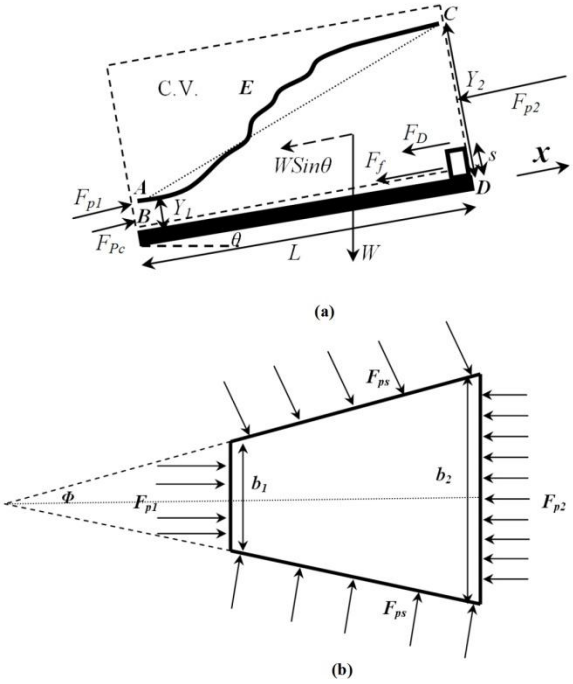


Fig. 1 (a) Acting forces on hydraulic jump on an adverse slope, (b) Plan view of acting forces on diverging hydraulic jump

Hydraulic jump on an adverse slope is an unstable phenomenon which causes some complexities in controlling of the jump which is the core of many reports published by different investigators in past decades. Okada and Aki (1959) carried several experiments of a stabilized hydraulic jump on an adverse sloping bed and finished on a reverse one. The experiments of Okada and Aki (1959) were carried out for approaching Froude numbers in the range of 9 to 13 which was outside the normal design range of conventional stilling basins. Rajaratnam (1966) classified the hydraulic jump on an adverse slope as F-type and concluded that ''it is almost impossible to establish a complete jump on the reverse slope,''. Hartung and Csallner (1967) indicated that an adverse transitional section at the downstream of a bucket spillway could be used to stabilize the hydraulic jump at low Froude numbers. They used a roughened bed in their experiments. Ohashi et al. (1973) studied the hydraulic jump which was located entirely on the adverse basin and confirmed its instability. McCorquodale and Mohamed (1994) carried out a set of experiments where both the toe and end of the hydraulic jump were located inside the adverse bed. They reported that the jump would be established at Froude numbers larger than 9 and would require continuous adjustment of tailwater to maintain a stationary position at Froude numbers less than about 4. Pagliara and Peruginelli (2000) concluded from their experiments that the presence of an end sill at the end of adverse stilling basins would stabilize the position of the hydraulic jump only for Froude numbers greater than 7.5. Baines and Whitehead (2003) reported that the jump was stable at downslope and was unstable in adverse basins regardless of Froude number and slope quantities. According to studies conducted by Defina et al. (2008) on the hydraulic jump at an adverse prismatic basin, the existence of a stable hydraulic jump is related to the presence of friction at large flow boundaries.

This paper reports the derivation of a theoretical equation, using momentum equation, to achieve the minimum Froude number of incoming flow in an adverse stilling basin where a stable hydraulic jump could be attained. The effects of end sill, divergence of side walls, boundary friction and water weight on adverse basin were also considered to develop partial required conditions for a stable hydraulic jump.

2. Estimation of minimum upstream Froude number for establishing hydraulic jump on an adverse slope

Establishing a hydraulic jump on an adverse slope requires overcoming the water weight on stilling basin which needs a minimum hydrodynamic force induced by incoming supercritical flow.

a) Hydraulic jump on prismatic channels: For free hydraulic jump on adverse basins, forces influencing the control volume, as shown in Figs. a. 1 and b. 1 are:

- ✓ Forces due to hydrostatic pressure distribution at the beginning and the end of a hydraulic jump are F_{p1}, F_{p2}, respectively,
- ✓ Force due to deviation from hydrostatic pressure distribution at the beginning of the hydraulic jump is F_{Pc},
- ✓ Weight of the water enclosed between the approach and sequent depths is W,
- ✓ The drag force exerted by end sill is F_D,
- ✓ The lateral forces resulted from side walls are F_{ps}, and,
- ✓ Friction force is F_f.

Assuming the hydrostatic pressure distribution at the toe and end of a hydraulic jump ($F_{Pc}=0$), overlooking from the normal acceleration, the air entrainment, and the effect of friction force on the hydraulic jump ($F_f=0$), the momentum equation in an adverse sloping prismatic rectangular channel can be written as:

$$\sum F_x = F_{p1} - \left(F_{p2} + W\sin\theta\right) = \rho bq\left(\beta_2 V_2 - \beta_1 V_1\right) \quad (1)$$

in which, $F_{p1} = \gamma b\frac{Y_1^2}{2}\cos\theta$, $F_{p2} = \gamma b\frac{Y_2^2}{2}\cos\theta, W = \gamma kbL\left(\frac{Y_1+Y_2}{2}\right)$, b is the width of the basin, ρ is the density of water,

γ is the specific weight of water, Y_1, Y_2 are the flow depths, V_1, V_2 are the flow velocities, θ is the angle of the bed slope, L is the length of stilling basin, q is the flow discharge per unit width of the rectangular basin, β_1, β_2 are the momentum correction coefficients at the beginning and the end of hydraulic jump, respectively. k is a coefficient for determining the weight of water in the control volume (W_{ABCDE}) to the weight of the water considering linear profile for the hydraulic jump (W_{ABCD}), (see Fig. a. 1). McCorquodale and Mohamed (1994) measured the velocity profile in the jet near supercritical section and approximated β_1 with 1.03. Okada and Aki (1959) proposed $k = 1.29 - 1.49$ for slopes of $-\frac{1}{4}, -\frac{1}{5}, -\frac{1}{6}$ while McCorquodale and Mohamed (1994) $k = 0.78 - 1.38$ for $0, -\frac{1}{5}, -\frac{1}{6}, -\frac{1}{10}$. However, Pagliara and Peruginelli (2000) concluded that the k values do not depend on the bed slope or the approaching Froude number and found a constant value of $k = 1.06$ from their experimental data. They proposed $\beta_2 = 1$ for the velocity distribution at the end of a hydraulic jump. Substituting the above relationships in Eq. (1) results:

$$\frac{q^2}{g} = \left(\frac{4Y_1Y_2}{4Y_1 - (4\beta_1)Y_2}\right)\left(\left(\frac{Y_1^2}{2} - \frac{Y_2^2}{2}\right)\cos\theta - kL\left(\frac{Y_1 + Y_2}{2}\right)\sin\theta\right) \quad (2)$$

Introducing $m = \frac{Y_2}{Y_1}$, the upstream Froude number for hydraulic jump on the sloping prismatic rectangular channel $Fr_1 = \frac{q}{\sqrt{gY_1^3\cos\theta}}$, and assuming $\beta_1 = 1$, $k = 1$, one obtains:

$$Fr_1^2 = \frac{m\left(1 - m^2 - \frac{L\tan\theta}{Y_1}(1+m)\right)}{2(1-m)} \quad (3)$$

where, $m = \frac{Y_2}{Y_1}$ which could be equal or less than the sequent depth ratio depending on the length of the stilling basin. Equation (3) is a general estimation of a complete free jump in the basin where upstream Froude numbers can take any values.

Figure 2 depicts the variation of the upstream Froude number with flow characteristics and bed dimensions from which the following results can be drawn:

1- The ratio of flow depth decreases with the length or bed slope, for a certain value of upstream Froude number,

2- Figure 2 indicates that a minimum value of upstream Froude number ($Fr_{1\min}$) is required to establish the free hydraulic jump on an adverse bed despite of classical hydraulic jump on a horizontal bed.

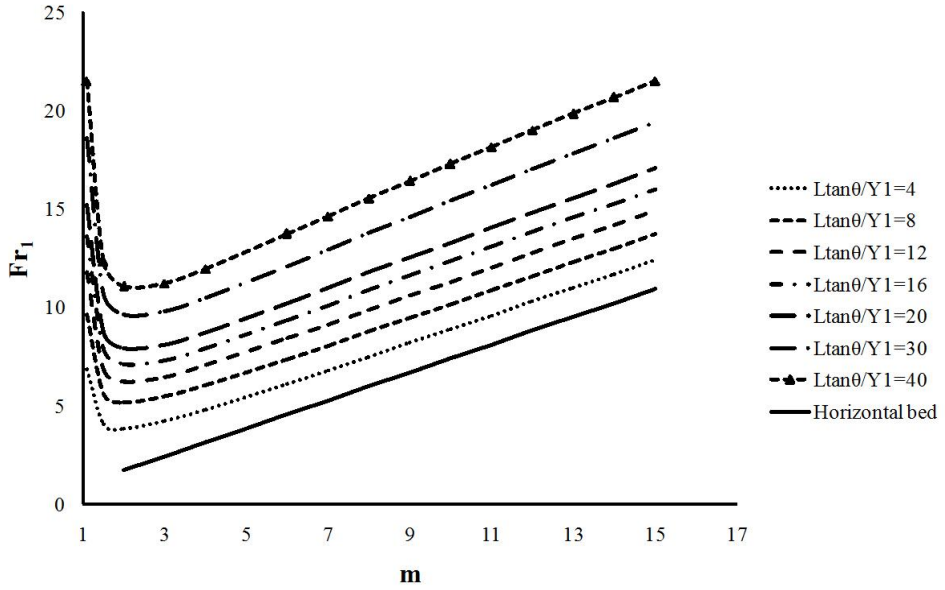


Fig. 2 Determination of sequent depths in hydraulic jump on an adverse slope

Figure 3 compares the minimum Froude numbers determined from Fig. 2 which is based on experimental data from several researchers. It is found that the hydraulic jump at the lesser Froude number is more submerged.

Using an end sill, drag force (F_D) will influence the control volume, as shown in Fig. a. 1. The drag force exerted by end sill is defined as:

$$F_D = \frac{1}{2}\rho C_D b s V_1^2 \qquad (4)$$

where, $V_1 = \frac{q}{Y_1}$, s is the sill height, and C_D is the drag coefficient. Ohatsu et al. (1991) proposed the following relationship for estimating the drag coefficient due to an end sill at the distance x_s from the toe of a classical hydraulic jump which is assumed valid for hydraulic jump on an adverse slope:

$$C_D = 0.71 - 0.85\left(\frac{x_s}{5.5Y_2}\right) \qquad (5)$$

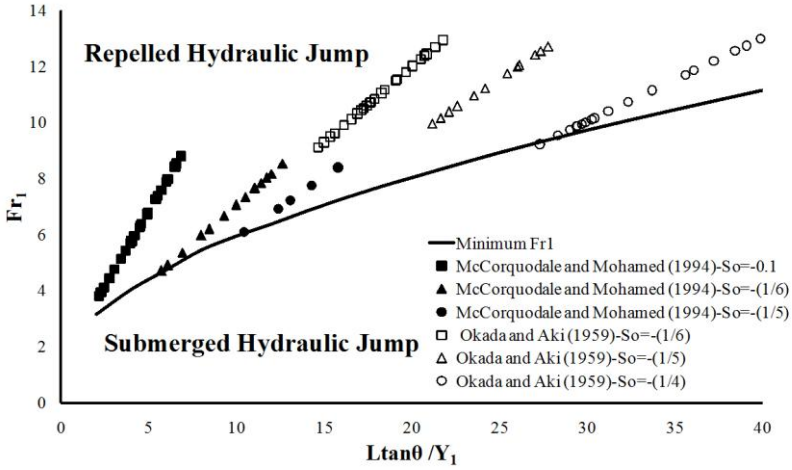


Fig. 3 The minimum values of upstream Froude numbers for developing free hydraulic jump on adverse bed

Substituting the drag force in Eq. (1) and following a similar procedure, we could obtain the following relationship for estimating the upstream Froude number required to stabilize the hydraulic jump on an adverse basin with an end sill:

$$Fr_1^2 = \frac{m\left(1-m^2-\frac{L\tan\theta}{Y_1}(1+m)\right)}{2(1-m)+C_D Sm} \qquad (6)$$

where $S=\frac{s}{Y_1}$. Figure 4 shows the effect of end sill height on developing hydraulic jump on an adverse slope for $\frac{L\tan\theta}{Y_1}=15$. It could be observed that the required upstream Froude number increases with the height of the end sill. Moreover, the ratio of flow depths decrease with the sill height, for certain values of upstream Froude number, length and bed slope, respective

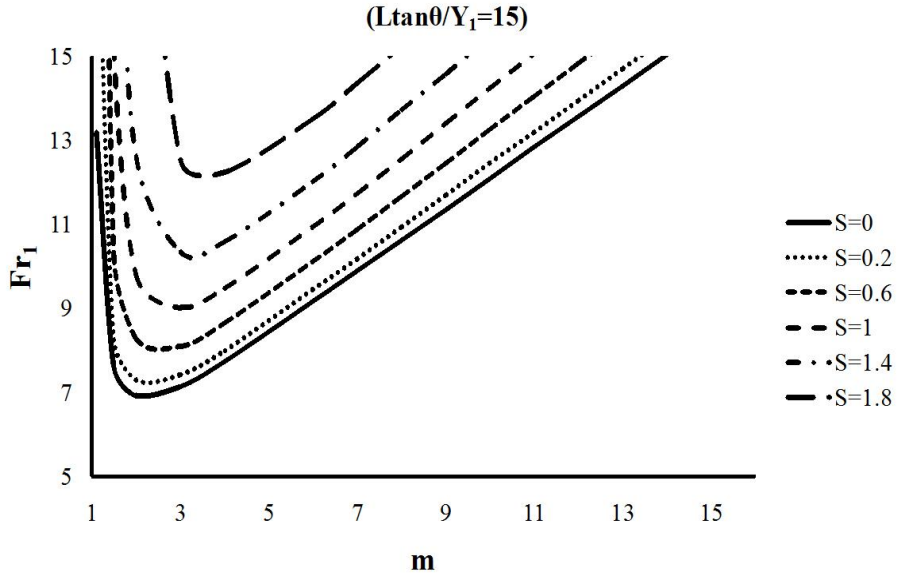


Fig. 4 Effects of end sill height on minimum values of upstream Froude numbers and sequent depths $(\frac{L\tan\theta}{Y_1}=15)$

b) Hydraulic jump on nonprismatic channels: For nonprismatic stilling basins with diverging walls, the lateral forces resulting from side walls (F_{ps}), influence the control volume, which are the reactions of pressure distribution in the *x*-direction (F_{psx}) (see Fig. b. 1). Kasi et al. (2011) investigated the diverging hydraulic jump with adverse bed. They compared linear, parabolic and elliptic profiles and recommended that the parabolic profile is more accordant with the experimental data. They have suggested the following relationship for the profile of hydraulic jump on an adverse slope in diverging rectangular basins:

$$Y(x)=Y_1+(Y_2-Y_1)\left[2\left(\frac{x}{L}\right)-\left(\frac{x}{L}\right)^2\right] \qquad (7)$$

Integration of Eq. (7) result:

$$WSin\theta = \gamma \sin\theta \int_{x=0}^{x=L} b(x).Ydx = \gamma \sin\theta \left[Lb_1\left(\frac{1}{3}Y_1 + \frac{2}{3}Y_2\right) + (b_2 - b_1)L\left(\frac{1}{12}Y_1 + \frac{5}{12}Y_2\right)\right] \quad (8)$$

where ϕ is the diverging angle of the basin section. Substituting the lateral pressure force into Eq. (1), gives the following equation for

Assuming hydrostatic pressure distribution yields:

$$F_{psx} = 2F_{ps}\sin\phi = 2\gamma\sin\phi \int_{x=0}^{x=L} \frac{Y^2}{2}dx = 2\gamma L\sin\phi\left(\frac{1}{10}Y_1^2 + \frac{4}{15}Y_2^2 + \frac{2}{15}Y_1Y_2\right) \quad (9)$$

estimating the required upstream Froude number in this case:

$$Fr_1^2 = \frac{m}{B\left(1-\frac{m}{B}\right)}\left[\frac{1}{2} + \frac{2\kappa L\sin\phi}{Y_1\cos\theta}\left(\frac{1}{10} + \frac{4}{15}m^2 + \frac{2}{15}m\right) - \frac{m^2}{2B} - \frac{L\tan\theta}{Y_1}\left(\frac{1}{3} + \frac{2}{3}m + \left(\frac{1-B}{B}\right)\left(\frac{1}{12} + \frac{5}{12}m\right)\right)\right] \quad (10)$$

in which $\kappa = \frac{Y_1}{b_1}$, $B = \frac{b_1}{b_2} = \frac{1}{1 + 2\kappa\frac{L\tan\theta}{Y_1}}$ ..

The effects of diverging section on flow depths ratios and upstream Froude number for $\frac{L\tan\theta}{Y_1} = 15$, $\kappa = 0.072$ is shown in Fig. 5.

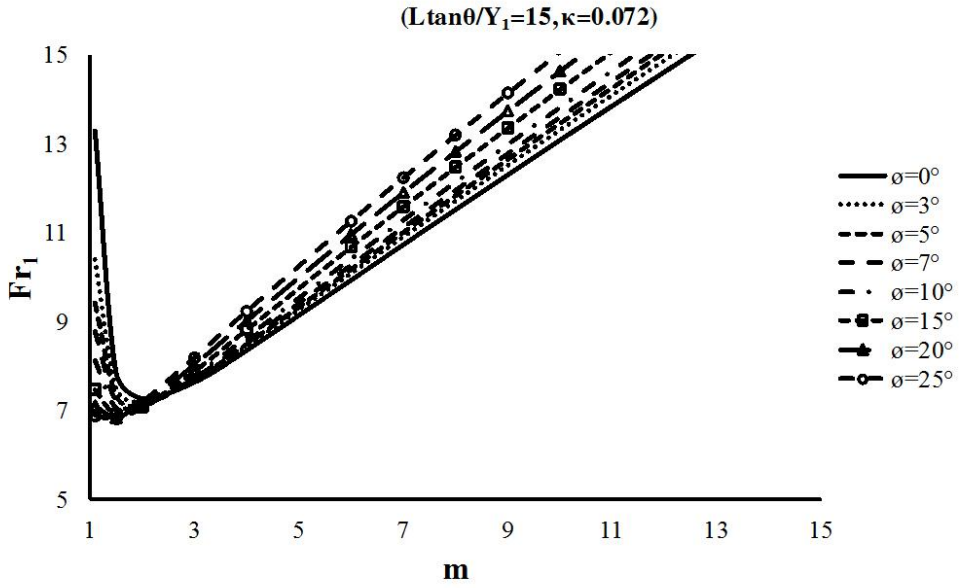


Fig. 5 Effects of divergence basin angle on minimum values of upstream Froude numbers and sequent depths $\left(\frac{L\tan\theta}{Y_1} = 15, \kappa = 0.072\right)$

It can be seen that the threshold value of Froude number for free hydraulic jump will decrease as the angle of divergence increases. This is the result of increased lateral pressure forces on decreasing the contribution of hydrostatic pressure force at the end of the stilling basin. It is evident that the depth ratio from the end to the toe will decrease in the diverging section. Interpolating the minimum threshold values from Eq. (10), gives the following regression relationship for estimating the minimum values of upstream Froude numbers related to the geometry of basin:

$$Fr_{1(\min)} = 1.564\left(\frac{L\tan\theta}{Y_1}\right)^{0.522} + 0.878e^{-0.026\phi} \tag{11}$$

It is noted that the Eq. (11) only represents the requirements to establish a free hydraulic jump on an adverse slope.

3. Theoretical criterion for establishing a hydraulic jump on an adverse slope

The stability criterion defined by Defina et al. (2008) was presented for adverse slopes with bed slopes less than -4%. They have overlooked the effects of water weight on sequent depths and its role on jump instability. This negligence doubts the reliability of their recommendations for larger slopes. By referring to Fig. a. 6 and using the momentum equation for a control volume enclosed between sequent depths at steady state conditions, one can obtain:

$$M_1 = M_2 + \frac{W\sin\theta}{\gamma b} + \frac{F_f}{\gamma b} \tag{12}$$

where M_1, M_2 are the specific forces at sections (1) and (2), respectively. In Fig. b. 6 the hydraulic jump dispositions and hence sequent depths in sections (1) and (2) are decreased to y_1'', y_2''.. Considering a unit control volume in both cases, the momentum equation is rewritten as:

$$M_1' = M_2' + \frac{W'\sin\theta}{\gamma b} + \frac{F_f'}{\gamma b} + f' \tag{13}$$

where $f' = \frac{F'}{\gamma b}$, and F' is the stabilizing force to restore the jump to its primary state. Differentiating Eqs. (12) and (13) on a small decrement, gives:

$$\frac{dM_1}{dx} = \frac{dM_2}{dx} + \frac{\sin\theta}{\gamma b}\left(\frac{dW}{dx}\right)_{|2} + f' \tag{14}$$

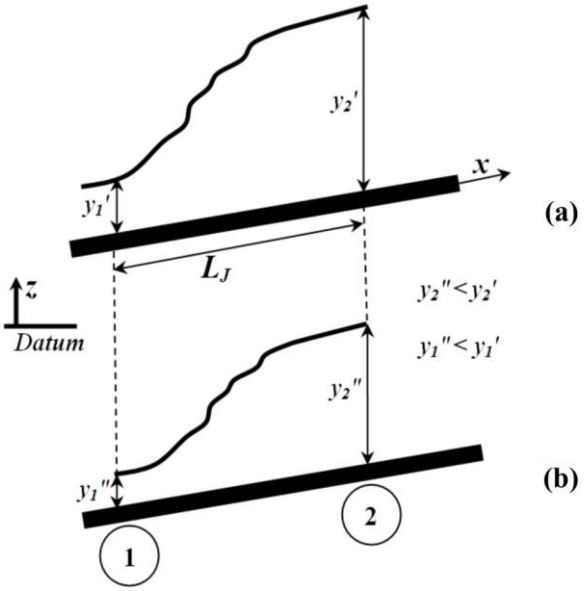


Fig. 6 Sketch of hydraulic jump on an adverse slope in a) stable stationary state, b) unstable stationary state

The hydraulic jump is defined stable if it slightly displaces from its equilibrium location and returns to its initial position (Baines and Whitehead, 2003). Therefore, a stabilizing force is required to achieve the stability of a hydraulic jump, (i.e. $f' > 0$):

$$f' = \frac{dM_1}{dx} - \frac{dM_2}{dx} - \frac{\sin\theta}{\gamma b}\left(\frac{dW}{dx}\right)_{|2} > 0 \rightarrow \frac{dM_1}{dx} > \frac{dM_2}{dx} + \frac{\sin\theta}{\gamma b}\left(\frac{dW}{dx}\right)_{|2} \quad (15)$$

The specific force on a sloping bed can be defined as:

$$M = \frac{y'^2}{2}\cos^3\theta + \frac{q^2}{gy'\cos\theta} \quad (16)$$

Differentiating Eq. (16) and using the chain role gives:

$$\frac{dM}{dx} = \frac{dM}{dy'} \times \frac{dy'}{dx}, \frac{dM}{dy'} = \frac{d}{dy'}\left(\frac{y'^2}{2}\cos^3\theta + \frac{q^2}{gy'\cos\theta}\right) = y'\cos^3\theta - \frac{q^2}{gy'^2\cos\theta} \quad (17)$$

On the other hand, differentiation of Bernoulli's equation at the x direction, results in the following

$$H = z + y'\cos\theta + \frac{q^2}{2gy'^2\cos^2\theta} \rightarrow \frac{dH}{dx} = \frac{dz}{dx} + \frac{dy'}{dx}\left(\cos\theta - \frac{q^2}{gy'^3\cos^2\theta}\right) \quad (18)$$

where, $\frac{dz}{dx} = -\sin\theta$, $\frac{dH}{dx} = -S_f$. Byintroducing the friction slope from the Chézy formula $S_f = \frac{q^2}{C^2y^3} = \frac{q^2}{C^2y'^3\cos^3\theta}$, the Froude number on the sloping bed $Fr = \frac{q}{\sqrt{gy'^3\cos^4\theta}}$ and assuming a wide channel ($R \approx y$), yields that:

$$\frac{dy'}{dx} = -\frac{c^2\tan\theta + Fr^2}{c^2\left(1 - Fr^2\right)}, \frac{dM}{dx} = -y'\cos^3\theta\left(\frac{c^2\tan\theta + Fr^2}{c^2}\right) \quad (19)$$

where, $c = \frac{C}{\sqrt{g}}$ and C is the Chézy coefficient. Despite recommendations by Kasi et al. (2011), and to achieve a simplified relationship, the linear profile for a hydraulic jump on an adverse slope is assumed by the following formulas:

$$y = y_1 + \left(\frac{y_2 - y_1}{L_J}\right)x, W(x) = k\gamma b\int_0^x y dx \rightarrow \frac{1}{\gamma b}\left(\frac{dW}{dx}\right)_{|2} = -ky_2 = -ky_2'\cos\theta \quad (20)$$

It should be noted that the weight of water in the control volume decreases when moving to a new state and $\frac{dW}{dx} < 0$. Introducing the above relationships in Eq. (15) and using $Fr_2^2 = Fr_1^2\left(\frac{y_1}{y_2}\right)^3$, yields:

$$c^2 < \frac{\left(\left(\frac{y_2}{y_1}\right)^{-2} - 1\right)Fr_1^2\cos^3\theta}{-k\left(\frac{y_2}{y_1}\right)\sin\theta + \left(1 - \frac{y_2}{y_1}\right)\cos^2\theta.\sin\theta} \quad (21)$$

On the other hand, Pagliara and Peruginelli (2000) proposed the following relationship for estimating sequent depths of a hydraulic jump on an adverse slope at $0 \prec \tan\theta \prec 0.25$:

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8G_1^2} - 1\right), G = 3.317^{-1.518\tan\theta}Fr_1 \quad (22)$$

Combining the above equations will result in the required conditions to establish a stable hydraulic jump on adverse beds:

$$C = c\sqrt{g} \rightarrow C^2 < g\frac{\left(4\left(\sqrt{1+8G_1^2} - 1\right)^{-2} - 1\right)Fr_1^2\cos^3\theta}{-\frac{k\sin\theta}{2}\left(\sqrt{1+8G_1^2} - 1\right) + \left(1 - \frac{1}{2}\left(\sqrt{1+8G_1^2} - 1\right)\right)\cos^2\theta.\sin\theta} \quad (23)$$

Equation (23) indicates the role of bed friction on the stability of an upward hydraulic jump, previously mentioned by Defina et al. (2008). The equation dictates that the stability of a hydraulic jump on an adverse slope requires a maximum value of Chézy coefficient as a function of upstream Froude numbers and bed slopes. Figure 7 demonstrates the variation of threshold values of Chézy coefficient with the upstream Froude number and bed slope. It can be concluded that:

- ✓ The jump would return to its stationary position as the friction force becomes larger than its threshold values,
- ✓ As the bed slope decreases, the range of the Chézy coefficient for jump stability grows.
- ✓ For a given value of bed slope, hydraulic jump on an adverse slope at higher upstream Froude numbers will be more stable if the boundary roughness is lesser. This is verified by reports from McCorquodale and Mohamed (1994) and Pagliara and Peruginelli (2000).

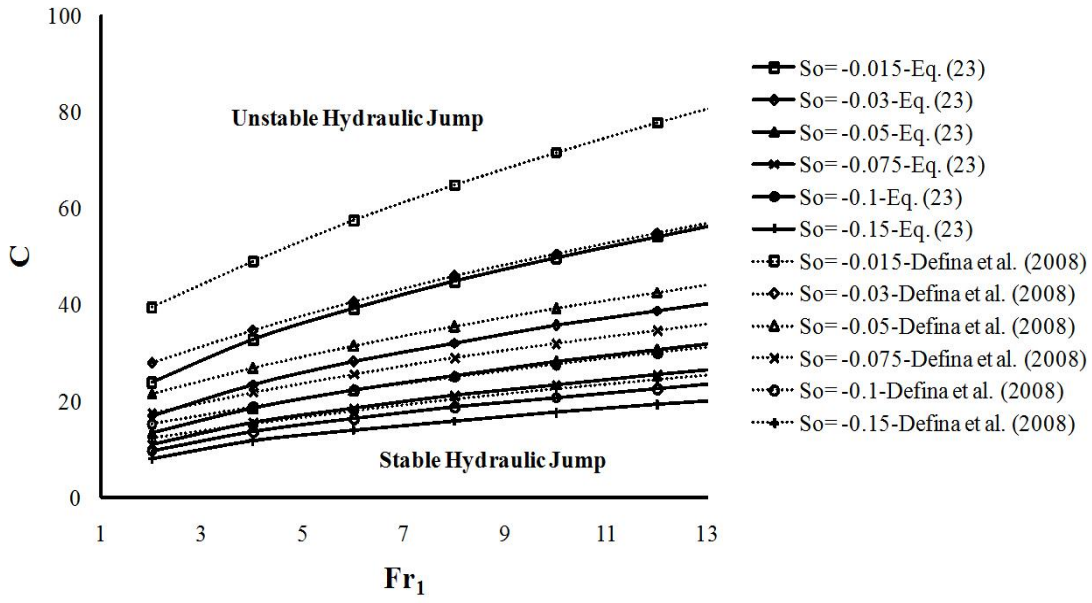


Fig. 7 Comparison between Eq. (23) and Defina et al. (2008) in estimating the maximum values of Chézy coefficient for stability of hydraulic jump on an adverse slope

Figure 7 compares the proposed criterion based on Eq. (23) and that of Defina et al. (2008). It can be seen that maximum values of the Chézy coefficient determined from Defina et al. (2008), would be greater than those calculated by Eq. (23). This can be attributed to overlooking of water weight on adverse basin.

Using Manning's equation, the minimum required roughness for stability of a hydraulic jump on an adverse slope downstream of a sluice gate, will be:

$$\xrightarrow{C=\frac{1}{n}y_1^{1/6}} n \geq \left[\left(C_c w\right)^{1/3} \frac{\cos^2\theta.\sin\theta\left(1-\frac{1}{2}\left(\sqrt{1+8G^2}-1\right)\right)-\frac{k\sin\theta}{2}\left(\sqrt{1+8G^2}-1\right)}{\left(2^{7/3}\left(\sqrt{1+8G^2}-1\right)^{-7/3}-1\right)Fr_1^2 g\cos^{8/3}\theta} \right]^{1/2} \tag{24}$$

where $C_c = \frac{y_1}{w}$ is the contraction coefficient. Figure 8 illustrates the variation of Manning's factor at the threshold state with the upstream Froude number and bed slope, assuming $C_c = 0.61$, and $w = 1.5cm$.

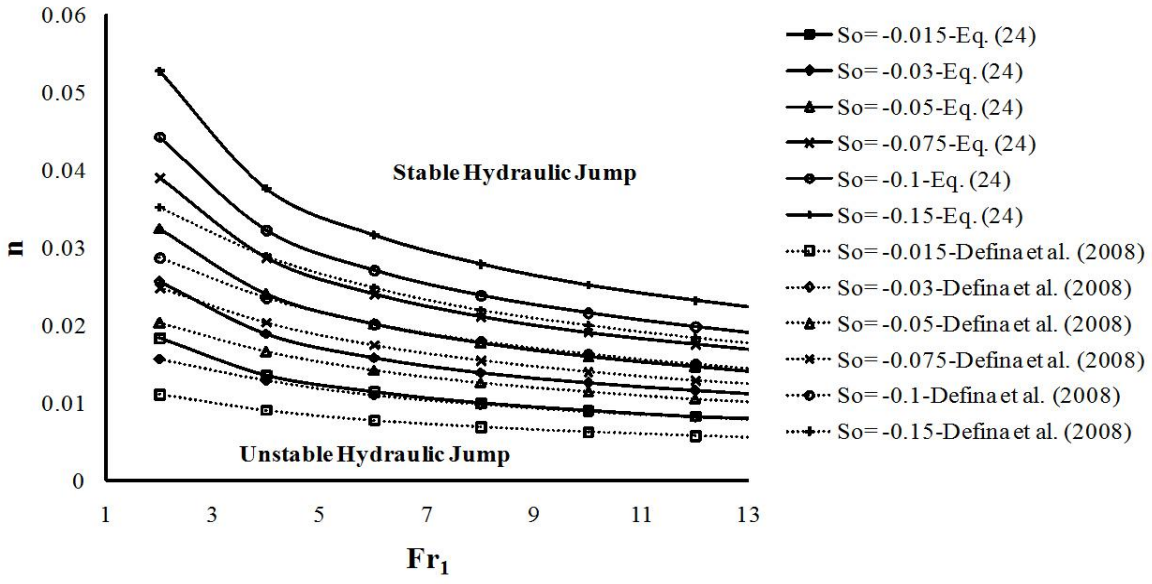


Fig. 8 Comparison between Eq. (24) and Defina et al. (2008) in estimating the minimum values of Manning's factor for stability of hydraulic jump on an adverse slope

These values were assumed for evaluating the minimum required bed roughness in a laboratory scale. Considering Manning's factor for concrete stilling basin (i.e. 0.014) and gate openings in a practical range, it is impossible to establish the free hydraulic jump on adverse bed in absence of any appurtenances. For this reason, presence of artificial roughness, sills or baffle blocks is required to stabilize the hydraulic jump on an adverse slope. The bed friction would be responsible for stability of a hydraulic jump on an adverse slope at a minimum upstream Froude number as shown in Eq. (11). Consequently, the minimum Froude number is not enough for stability of a hydraulic jump on an adverse slope. It should be noted that as the bed slope increases, the discrepancy among the values obtained from Defina et al. (2008)'s equation and Eq. (23) become larger. For example in slope -0.15 and initial Froude number of 9 for a hydraulic jump, Defina et al. (2008)'s equation, attains the minimum Manning coefficient needed for a stable jump about 21 percent less than recommended method.

From Fig. 8, it can be concluded that neglecting the influence of water weight on adverse bed in Defina et al. (2008)'s equation, will mislead in recognition of the stability limits of a hydraulic jump on an adverse slope. However, Eq. (23) can resolve this defect.

In case of a diverging hydraulic jump on an adverse slope, if induced lateral pressure force by side walls is introduced into Eq. (15), the following relationship will be obtained:

$$C^2 < g \frac{\left(\frac{b_2}{b_1}.\frac{y_2}{y_1}Fr_2^2 - Fr_1^2\right)\cos^4\theta}{\left[\cos^3\theta.\tan\phi\left(\left(\frac{y_2}{y_1}\right)^2\left(\frac{y_1}{b_1}\right)\left(1-2Fr_2^2\right)-\frac{y_1}{b_1}\left(1-2Fr_1^2\right)\right)+\cos^3\theta.\sin\theta\left(1-\frac{b_2}{b_1}\frac{y_2}{y_1}\right)+\sin\phi\left(\frac{y_2}{y_1}\right)^2\left(\frac{y_1}{b_1}\right)-\frac{b_2}{b_1}\frac{y_2}{y_1}\cos\theta.\sin\theta\right]} \quad (25)$$

in which, $\frac{b_2}{b_1} = 1 + 2\frac{L_J}{y_1}\frac{y_1}{b_1}\tan\phi$ and $Fr_2^2 = Fr_1^2\left(\frac{y_1}{y_2}\right)^3\left(\frac{b_1}{b_2}\right)^2$. Kasi et al. [9] proposed the following relationships for estimating the sequent depths and the length of a diverging hydraulic jump on adverse bed from experimental data:

$$\frac{y_2}{y_1} = 0.309(1-\tan\theta)^{3.542}.(1+\phi)^{-0.1197}.(Fr_1)^{0.5175}.\left(\frac{L_J}{y_1}\right)^{.5444} + 1.4396$$
$$\frac{L_J}{y_1} = 2.9607(1-\tan\theta)^{3.7214}.(1+\phi)^{-0.0774}.(Fr_1)^{1.4997} \quad (26)$$

Figure 9 shows the effect of the divergence angle on the stability of a hydraulic jump for two different bed slopes and $\frac{y_1}{b_1} = 0.072$. It can be observed that the minimum required bed friction to stabilize the hydraulic jump on an adverse slope will decrease as the divergence angle increases. For example at a slope equal to -0.05, it can be observed from Fig. 9 that an increase of the divergence angle of the basin from 0 to 1.5 degrees at a initial hydraulic jump Froude number of 0.9 causes an approximate 37% rise in the maximum possible Chezy coefficient for a stable hydraulic jump. Therefore, the establishment of a stable hydraulic jump on an adverse slope can be achieved in nonprismatic basins. Figure 9 was depicted for a certain condition where the length of the adverse basin equals to the length of the hydraulic jump. One can employ Eq. (26) to observe any adverse basins with different arbitrary dimensions. It should be noted that the minimum friction necessary for stability of a hydraulic jump from Figs. 7,8 and 9 for specific values of gate opening and initial diversion basin width is depicted and lacks practical purposes. Despite this proposed Eqs. (23), (24) and (25) due to their theoretical bases, can be used in general design of adverse basins and for reliability of the stability of a hydraulic jump.

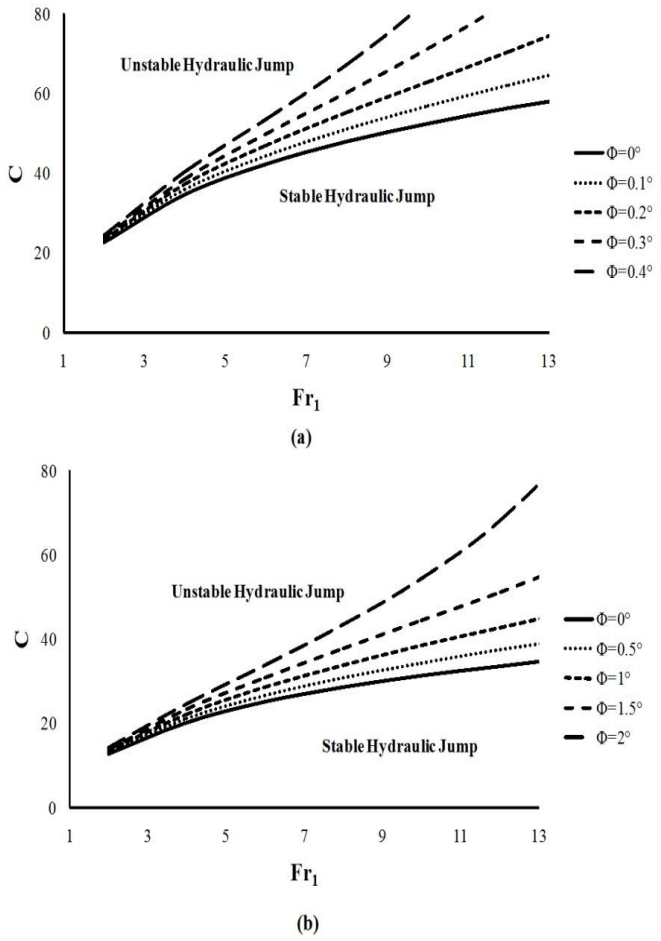


Fig. 9 Effect of divergence angle and bed slope on stability of hydraulic jump on an adverse slope a) $S_o = -0.015$, b) $S_o = -0.05$

4. Conclusions

In this research, a theoretical equation for estimating the minimum upstream Froude number to establish the free hydraulic jump on an adverse bed was presented. The following conclusions may be drawn:

- ✓ The required upstream Froude number increases when the height of end sill increases. Also, the threshold value of Froude number for free hydraulic jump would be decreased as the angle of divergence increases.
- ✓ A theoretical relation was developed to estimate the minimum values of upstream Froude numbers related to basin geometries.
- ✓ Considering the Manning's factor for concrete stilling basin and gate opening in practical range, it is impossible to establish the free hydraulic jump on an adverse bed in the absence of any appurtenances.
- ✓ Some deviations were observed in estimating the stability criterions from both proposed method and Defina et al. (2008), especially in large bed slopes.
- ✓ The minimum required bed friction to stabilize the hydraulic jump on an adverse slope decreases when the divergence angle increases.
- ✓ The establishment of stable hydraulic jump on an adverse slope could be achieved in the nonprismatic basins.
- ✓

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Notation

The following symbols are used in this paper:

F_{p1} Force due to hydrostatic pressure distribution at the beginning of hydraulic jump

F_{p2} Force due to hydrostatic pressure distribution at the end of hydraulic jump

F_{Pc} Force due to deviation from hydrostatic pressure distribution at the beginning of hydraulic jump

W Weight of the water enclosed between the approach and sequent depths

F_f Friction force

θ Angle of the basin slope

ρ Density of water

b Width of basin

q Flow discharge per unit width of the rectangular basin

β_2 Momentum correction coefficients at the end of hydraulic jump

β_1 Momentum correction coefficients at the beginning of hydraulic jump

γ Specific weight of water

Y_1 Flow depth at the beginning of hydraulic jump

Y_2 Flow depth at the end of stilling basin

k Shape factor for hydraulic jump on an adverse slope

L Length of basin

m Ratio of flow depths at the beginning and the end of hydraulic jump

g Acceleration due to the gravity

Fr_1 Upstream Froude number

$Fr_{1\min}$ Minimum value of upstream Froude number for establishing the free hydraulic jump on adverse bed

F_D Drag force

C_D Drag coefficient

s Sill height

F_{ps} Lateral pressure force

ϕ Expending angle of basin in degree

M_1 Specific force at the beginning of hydraulic jump

M_2 Specific force at the end of hydraulic jump

F' Stabilizing force of moved jump to the primal state

y_1' Orthogonal depth at the beginning of hydraulic jump

y_2' Orthogonal depth at the end of hydraulic jump

H Total head

z Bed elevation

S_f Friction slope

C Chézy coefficient

R Hydraulic radius

n Manning's factor

C_c Contraction coefficient of sluice gate

w Gate opening

L_J Length of hydraulic jump

S_o Basin slope