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Developing Optimal Operating Reservoir Rule-curves in Drought Periods

Soroosh Alahdin[1]
Hamid Reza Ghafouri[2]

Abstract

In this study, a simulation-optimization model is developed for deriving operational rule-curves in drought periods. To each reservoir, two rule-curves with adjustable monthly levels are introduced dividing the reservoir capacity into three zones between the normal water level and minimum operation level. To each zone of the reservoir and for each month of the year a hedging coefficient is introduced that determines the release from the reservoir. Accordingly, an optimization problem is developed in which the objective is the minimization of water demands deficits in drought and the decision variables are the rule-curves levels and hedging coefficients. For optimization, a genetic algorithm equipped with a self-adaptive constraint handling strategy is used. To evaluate the objective function and constraints violations, the flexible and widely-used WEAP (Water Evaluation and Planning) simulation model is exploited and coupled with the optimization solver. The model is then applied to the Zohreh three-reservoir system in the southwest of Iran and compared to the Standard Operation Policy (SOP). According to the sustainability indices for the system operated in drought, the obtained operating rule-curves are found significantly superior to the SOP. As a result of applying the rule-curves, the modified shortage index (MSI) and vulnerability (extent) of the system are respectively improved by 22% and 28% compared to the SOP. Consequently, the developed policy application resulted in longer periods of deficit (but less severe) as shown by decrease in reliability (5%) and resilience (40%) indices.

Keywords: Operating policy, Rule-Curve, Sustainability Indices, Optimization Model, Hedging.

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[1] Ph.D. Candidate, Department of Civil Engineering, Engineering Faculty, Shahid Chamran University of Ahvaz, Ahvaz, Iran, s-alahdin@phdstu.scu.ac.ir (**Corresponding author**)
[2] Professor, Department of Civil Engineering, Engineering Faculty, Shahid Chamran University of Ahvaz, Ahvaz, Iran.

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1. Introduction

Handling the operational challenges has been always a serious concern for reservoirs operators, but when a water resources system is suffering from water scarcity, demands are more strongly competing for water and the system operation becomes even more complicated and challenging.

Application of simulation and optimization models has shown great advances in planning and management of water resources problems. Currently, there are several softwares like HEC3, HEC5, HEC-ResSim, MIKEBASIN, RIBASIM, WATHNET, OASIS, ARSP, CalSim, RiverWare, MODSIM, WRAP, SWD SUPER and WEAP that can simulate operation of a reservoir system with its all components in details. A simulation model can evaluate the system performance for any decision on the reservoirs releases and operations over a certain time period and return the values of objective functions and constraint violations introduced to the system.

A reservoir system is operated following prescriptions on the system releases as a function of time and available water. These prescriptions or more precisely rule-curves are traditionally derived through intensive simulation techniques [1], but to derive optimum operating ones for a multi-reservoir system, it is required to develop and solve optimization models. The optimization systematically changes the operational parameters and the simulation model evaluates the effects of each change on the system operation over a long period. This process continues until the best feasible performance of the reservoir system is achieved. This is the main skeleton of every simulation-optimization model that has recently become very popular and well-developed for solving a variety of water resources problems. In this context, many researchers have tried to cope with the challenges associated with the operation of reservoir systems in arid and semi-arid areas as well as during drought periods. The existing approaches are different in concept ranging from simplified rules of thumb to sophisticated model-based operating rules.

To minimize the impacts of a drought event, water is stored in the system for future even if enough water exists to satisfy the current demands. The purpose of a hedging decision in a reservoir operation is to lessen the burden of water scarcity in the future by distributing the demands' deficits over an extended period of operation. In general, hedging rules can be categorized into two main groups of continuous and discrete methods. Through the continuous approach, the decisions on the amount of water to be rationed are mathematically represented by a linear, nonlinear or fuzzy function [2-13]. Whereas, in the discrete methods the decisions are made based on the discrete representation of the reservoir and water resources conditions [1, 14-17]. Many investigations with the aid of simulation and optimization models were published to extend and modify these approaches. These studies put forth and answered two key questions of, "when to start rationing" and "how much rationing". However, this issue is highly case-dependent and needs a thoughtful insight into the system’s objectives, limitations, hydrology and other specifications.

An important concern in deriving hedging rules is the employment of proper solvers. The applied methods are the mixed integer programming [1-2, 14-15], dynamic programming [10], Markov Model [6, 7] and recently, heuristic nature-inspired algorithms [8-9, 11-12, 16-18]. For the sake of dimensionality problems associated with the large-scale multi-reservoir systems, some investigations were inevitably restricted to single-reservoir case studies or short-term time-series [2-10, 14-15, 19].

Some studies were carried out based on surrogate simulation models rather than simulating the system with all details. For example, to make the process of simulation-optimization more efficient Dariane et al., [5], trained and substituted an artificial neural network for the system simulation and Guo et al., [16], used an equivalent reservoir for a multi-reservoir system and

assumed constant weighting factors for different types of water supplies. However, issues like the interaction between the reservoirs and other components of the system as well as the role of long-term data encompassing different hydrological conditions are very significant in deriving the system operating rules. Paying attention to these concerns, Taghian et al., [17], developed a comprehensive long-term simulation model based on the ARSP for a three-reservoir system facing with severe droughts. All water demands were also simulated in the model using the network flow scheme. They proposed a discrete hedging rule by hybridizing a genetic algorithm for hedging optimization and a linear programming model for allocation water to downstream demands. In this context, the present study aims at developing a complete simulation-optimization model for multi-reservoir systems in drought periods with some modifications. The powerful, comprehensive and easy-to-use simulation software, WEAP is used to serve as objective function and constraint values provider. Then, the problem is formulated as a constrained nonlinear optimization problem and is solved using a self-adaptive Genetic Algorithm (GA). The model is applied to a three-reservoir case study in Iran, and the results are discussed and compared with those obtained by the SOP.

2. Case study and problem statement

As a case study, drought-stricken Zohreh river system located in the south-western of Iran with an area of 15460 squared kilometer is studied. The schematic of the system is shown in Figure 1. The system comprises of three reservoirs, eight input stream flows, thirteen irrigation networks, three public demands and two minimum flows. The conservation storage volumes for Kosar, Chamshir, and Kheirabad reservoirs are respectively, 418, 1862, and 105 million cubic meters (mcm) as presented in Table 1 in details.

Table 1. Properties of hydropower plants

Properties of dam	Chamshir	Kosar	Kheirabad
Normal level (meter above sea level)	598	625	259.6
Minimum level of operation (meter above sea level)	543.7	580	238
Volume storage in min. level (mcm)	454.5	74.2	0.93
Volume storage in normal level (mcm)	2316.7	492.8	106.3
Installed capacity (Mega Watt)	165	-	2.5
Plant factor (%)	25	-	-
Plant efficiency (%)	91	-	85
Maximum turbine flow (cms)	144.4	-	7.26

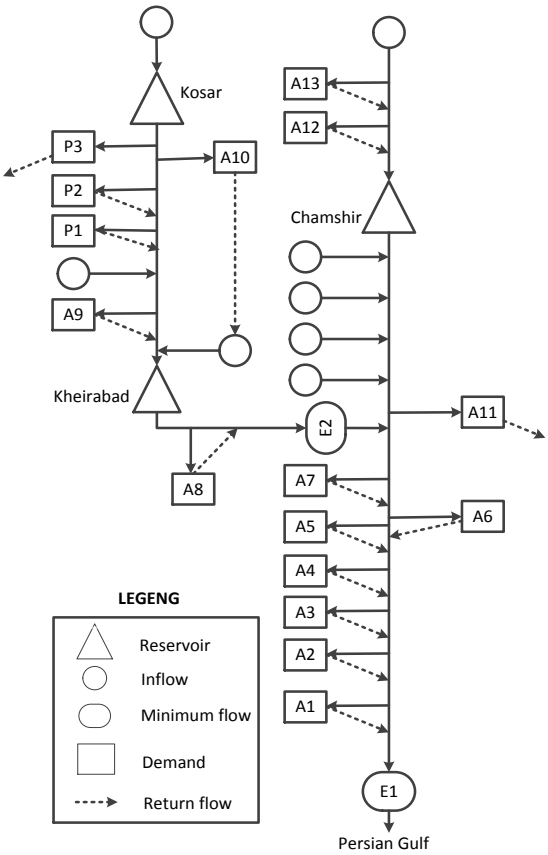


Figure 1. Schematic configuration of the water supply system

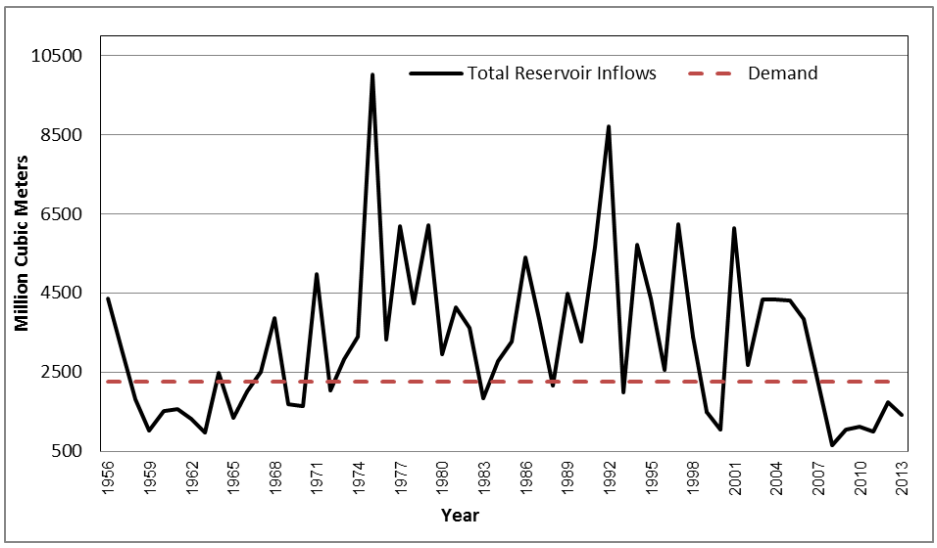


Figure 2. Total annual inflow and demand of the system

The total annual inflow to the system (sum of all reservoirs' inflows) and the total demand are depicted in Figure 2. The average value of total annual inflow is about 3247 mcm and the average value of the last seven years (2007-2013) is 1330 mcm. Comparison of these two average parameters reveals two long periods of drought in the initial and the last stages of the time-series. Target values for demands are given based on the planned water demand for the future horizon (2259 mcm), distributed as 73% for agriculture, 16% for the minimum flow, and 11% for public demands (Table 2).

Table 2. Characteristics of demands

Demand title	Type	Annual demand (mcm)
A1	Agriculture	20
A2	Agriculture	480
A3	Agriculture	60
A4	Agriculture	77
A5	Agriculture	49
A6	Agriculture	138
A7	Agriculture	114
A8	Agriculture	28
A9	Agriculture	38
A10	Agriculture	48
A11	Agriculture	365
A12	Agriculture	101
A13	Agriculture	132
P1	Public	32
P2	Public	10
P3	Public	210
E1	Minimum flow	315
E2	Minimum flow	41
SUM		2259

3. The simulation-optimization model

After construction of large-scale water storage projects, attention must be on improving the operational effectiveness and efficiency. To achieve this goal, a simulation-optimization model is developed in this study as a general framework for drought management (Figure 3). To this end, WEAP is embedded into a genetic algorithm and the hedging rule is included.

In addition to optimizing water allocation, this work aims at mitigating the drought consequences in the reservoir system. The objective is to alleviate the effects of water shortage by rationally distributing water in a longer horizon. In operation of this system, there are 74 decision variables including, two monthly rule-curves coordinates for each dam (Chamshir, Kheirabad and Kosar dams) and two hedging factors for the whole system. The objective function is to minimize the modified shortage index as Eq. 4. The performance constraints e.g. rationing rule-curve attributes are added in the constraint handling procedure of optimization algorithm. The physical constraints, mass balance, performance constraints and hedging rules are the main components that are included. To solve the problem, a constrained optimization algorithm is coupled with a standard simulation model as illustrated in Figure 3. All the physical constraints like the maximum dam release, reservoir storage, and channel capacity are handled in the general simulation model. To evaluate the long-term performance of reservoir system operation, 58-year (from 1955 to 2013) time-series of monthly inflows are used, resulting in a total of 696 months. In what follows, the simulation and optimization models are introduced in details and the results of case study optimization are presented and discussed.

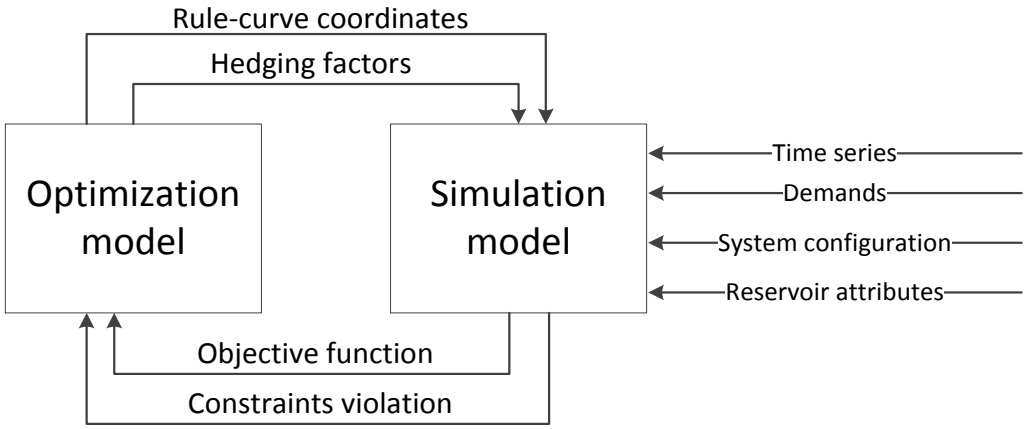


Figure 3. Conceptual model

3.1. Simulation Model (WEAP)

Reservoir simulation models are based on the mass balance in the system of reservoirs and demands. WEAP calculates water mass balance for every node and link in the system on a monthly time step. Implementing hedging rules in the simulation model adds three constraints (Equations 1, 2 and 3) to it. In drought periods, the inflow may not be adequate to keep the water storage level (S_t) on or above the first rule-curve if the target demand is satisfied. Releasing current available water from the reservoir to completely supply the target delivery may threat safe supply in future. Therefore, rationing is introduced to diminish the current reservoir release, and retain an adequate amount of water in storage for future consumption. When the initial reservoir storage level is between the two rule-curves (in the first hedging zone, $Zone_1$), the reservoir releases water to meet the first phase of hedging ($\alpha_1 Demand$, α_1 is the first hedging coefficient). For more severe droughts, when the beginning reservoir storage is below the second rule-curve (in the second hedging zone, $Zone_2$), less water is released from the reservoir to meet the second phase of hedging ($\alpha_2 Demand$, α_2 is the second hedging coefficient). The coordinates of the two rule-curves and hedging coefficients are decision variables which will be optimized in the simulation-optimization procedure.

$$if\ S_t \notin (Zone_1\ And\ Zone_2)\ Then\ Demand = Demand \times 100\% \quad \textbf{(1)}$$

$$if\ S_t \in Zone_1\ Then\ Demand = Demand \times \alpha_1 \quad \textbf{(2)}$$

$$if\ S_t \in Zone_2\ Then\ Demand = Demand \times \alpha_2 \quad \textbf{(3)}$$

3.2. Optimization model

The Genetic Algorithm starts with a randomly generated group of chromosomes known as the population. In each chromosome, there are as many genes as the number of decision variables. In this study the number of individuals in each generation was set to 104. In the first step, the costs and associated chromosomes are ranked from lowest to highest for the population. Survival of the fittest, is translated into discarding the chromosomes with higher costs. Therefore, the better chromosomes are selected to continue, while the rest are deleted. The selection rate (here 50%) is the fraction of the population that survives for the next step of reproduction.

Mating is reproduction of one or more youngsters from the parents selected in the pairing

procedure. The most common form of mating involves two parents producing two children. The uniform mating which is adopted here randomly assigns the gene from one parent to one offspring and the gene from the other parent to the other offspring.

In the next step, random mutations alter a certain percentage of the genes in the list of chromosomes. Mutation is the second way of a GA exploring a cost surface. It can introduce traits, not in the original population and keeps the GA from converging too fast before sampling the entire feasible space. The mutation rate was used for 2% of the genes of all the population.

After the occurrence of mutations, the costs associated with the offspring and mutated chromosomes are calculated. The process described above is iterated. The number of generations depends on whether an acceptable solution is reached or a number of *iterations* is exceeded. After a while all the chromosomes and associated costs would become the same if it were not for mutations. The flowchart in Figure 4 provides an overview of a constrained continuous GA. The primary difference with the usual algorithm is the constraint handling procedure which is discussed in the constraints section.

3.2.1. Objective Function

In the past for drought management, many objective functions were suggested and used, but the most used indices are SI and MSI. The shortage index (SI) was presented by the U.S. Army Hydrologic Engineering Center [20-21]. Because the SI index is the average of the "annual" deficit rate squared, to characterize the "monthly" fluctuations of the hydrologic time series, a modified shortage index (MSI) is defined as the following,

$$MSI = \frac{100}{n}\sum_{t=1}^{n}\left(\frac{TS_t}{TD_t}\right)^2 \quad \textbf{(4)}$$

Where TS_t is shortage in tth period; TD_t demand in tth period; and n number of periods [22]. For the energy production, the success or failure in each time step is evaluated based on the installed capacity of the hydropower plant. When the generated energy or the equivalent minimum volume of water discharge considering plant factor and minimum operation head, in that period is less than the installed capacity, a failure is counted. Then the deficit will be the difference between the turbine flow and the flow requirement to generate energy as much as installed capacity [23].

3.2.2. Constraints

To make the derived rule-curves meet the operational and real situation standards, two points must be noticed. First of all, the maximum difference between two consecutive operating reservoir storage targets (rule-curve coordinates) must not be more than a defined value [24]. This is because refilling and emptying a reservoir and generally fluctuations in reservoirs in the real world will follow a supply-demand pattern and its configuration could not be scattered. In mathematical expression, this could be written as,

$$|S_t - S_{t-1}| \leq S_{allowed} \quad \textbf{(5)}$$

Where S_t and S_{t-1} are corresponding to two consecutive operation rule-curve components (storage or trigger value here), and $S_{allowed}$ is the maximum allowable difference. Second, because at the end of the water year and the start of the new one, no significant hydrologic and demand changes are expected to take place in the basin, it is not logical that the difference between the first (S_1) and the last (S_{12}) components of a rule-curve be noticeable. Therefore, the second constraint would be written as follows,

$$(1-\varepsilon)\times S_1 \le S_{12} \le (1+\varepsilon)\times S_1 \quad \textbf{(6)}$$

Where, ε is a small number. To implement these constraints in the algorithm, a method based on the preference of feasible solutions over infeasible ones called self-adaptive is used. This method is implemented in the selection and ranking process in GA (Figure 4) and has the following characteristics: 1) as long as no feasible solution is found, the constraint violation is the rank determinant of the individuals; 2) once there is a combination of feasible and infeasible solutions in the population, feasible solutions will be ranked higher; and 3) the feasible solutions will be ranked based on their objective function values.

In addition, there are two other points that are considered in the algorithm. First, since no project can survive with less than 20% of its demand supplied and on the other side, there is no point in rationing with supply more than 80% of the demand, practical and operational view suggests that the hedging coefficient cannot be less than 20 or greater than 80 percent. Although in theory the allocation outside this range is possible. Second, lots of trial and errors with the simulation-optimization model have shown that two rule-curves for each dam reservoir may cross each other. Therefore, a mechanism must be devised to go around this issue. To do this in the optimization routine after mutation process, the values of the second hedging rule-curve are checked and forced to be less than the first phase (Figure 4).

4. Results

In order to show the efficiency and clarify algorithm's performance in multi-reservoir system mangement, in addition to the objective function value (MSI), other statistical measures like reliability, resilience, and vulnerability are also calculated. These criteria are defined as,

$$Reliability = \frac{number\ of\ satisfactory\ values}{total\ number\ of\ values} \quad \textbf{(7)}$$

$$Resilience = \frac{number\ of\ times\ a\ satisfactory\ value\ follows\ an\ unsatisfactory\ value}{total\ number\ of\ unsatisfactory\ values} \quad \textbf{(8)}$$

$$Vulnerability\ (extent) = expected\ extent\ given\ unsatisfactory\ values \quad \textbf{(9)}$$

The system operation performances for the SOP and the developed rule are compared in the next tables and figures; they display the long-term system performance and the annual system performance during recent failure years.

Table 3. Long-Term System Performance during the Period 1955–2013

Index	SOP	Optimum rule-curve
α_1	-	70.9
α_2	-	35.2
MSI	5.35	4.13
Average reliability	69.3	65.5
Average resilience	26.5	15.8
Average vulnerability extent	8.7	6.2

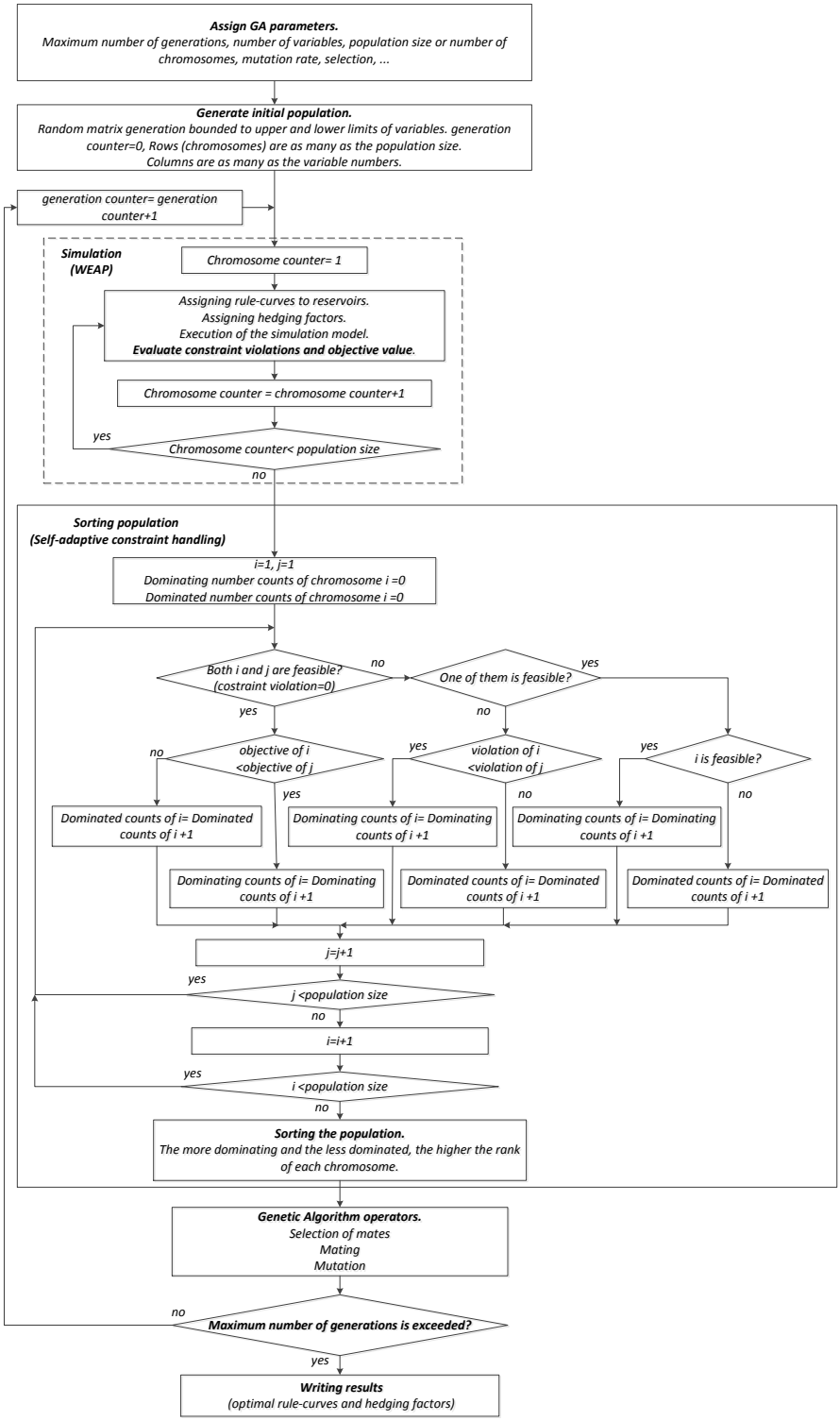


Figure 4. Flowchart of the proposed simulation-optimization model

The long-term performance criteria are given in Table 3. Comparing the two scenarios, it is indicated that total MSI value is improved by 22% in the proposed method. The distribution of severe droughts cause neighboring deficit periods join together, nevertheless resilience (which is defined as the number of recoveries) decreased from 26.5 to 15.8. Since the idea is to spread deficits to smoothen them in a longer horizon that could be more tolerable for the demands, the decrease in reliability (5%) is reasoned.

These points are rechecked and confirmed in Table 4 as well. The developed policy appears to be superior to the SOP, which is attributed to the inclusion of hedging rules. The hedging rule inherently tends to adjust the demand by rationing before and during the occurrence of severe shortages. Therefore, the SOP resulted in a better reliability than the value introduced by the model (Tables 3 and 4). The direct result of this kind of management is that the failures are less severe but more frequent and therefore, the resilience (the number of system restoration regarding to supplied water) is less for the planned policy compared to the SOP.

Table 4 gives long-term performance criteria for various demand groups. Almost the same pattern which was discussed in table 3 is dominating here. The agricultural demand's behavior which is accountable for 73% of the total system demand reveals the total demands' behavior. The difference in the performance criteria for the two management methods in public, environment and hydropower demands are negligible, since they all together are only 27% of the whole system demands and therefore model is biased towards fulfilling the agriculture demands.

Table 4. Long-term performance criteria for various demand groups

Demand type	Reliability (optimum)	Reliability (SOP)	resilience (optimum)	resilience (SOP)	vulnerability extent (optimum)	vulnerability extent (SOP)
Agriculture	68.1	75.1	7.5	23.7	4.8	7.4
Public	43.1	41.4	24.0	25.0	2.1	2.2
Environment	93.1	86.2	60.0	47.0	6.9	6.4

The annual objective function values for the recent drought (Table 5) show that the maximum value of MSI is reduced in three different groups of users. This point means that the proposed drought management is working well. The biggest difference in MSI values is in hydropower demand which is reduced more than 50 units in the 7-year drought. In this table, the hydropower objective function values show very informative changes during 2008 to 2012 for the two policies, that confirms the idea behind the optimization. In 2008, the optimum model used hedging and as a result the MSI value is higher (8.04> 7.06) but by saving water in this year, the system benefited in the next four years (21.05-29.17 for optimum and 81.72 to 82.25 for SOP).

Table 5. Objective function values for various users in recent drought

	Objective function value					
Year	Agriculture (optimum)	Agriculture (SOP)	Minimum flow (optimum)	Minimum flow (SOP)	Public (optimum)	Public (SOP)
2007	0.47	0.47	0.00	0.00	3.45	3.45
2008	6.72	1.48	0.05	0.27	15.32	16.45
2009	28.62	23.24	0.39	0.60	9.89	11.04
2010	23.92	20.11	0.00	0.17	1.49	2.56
2011	24.71	31.34	0.07	0.32	7.35	8.83
2012	17.98	5.03	0.22	0.35	5.34	6.17
2013	12.44	11.25	0.00	0.03	0.07	0.28
Max	28.62	31.34	0.39	0.60	15.32	16.45

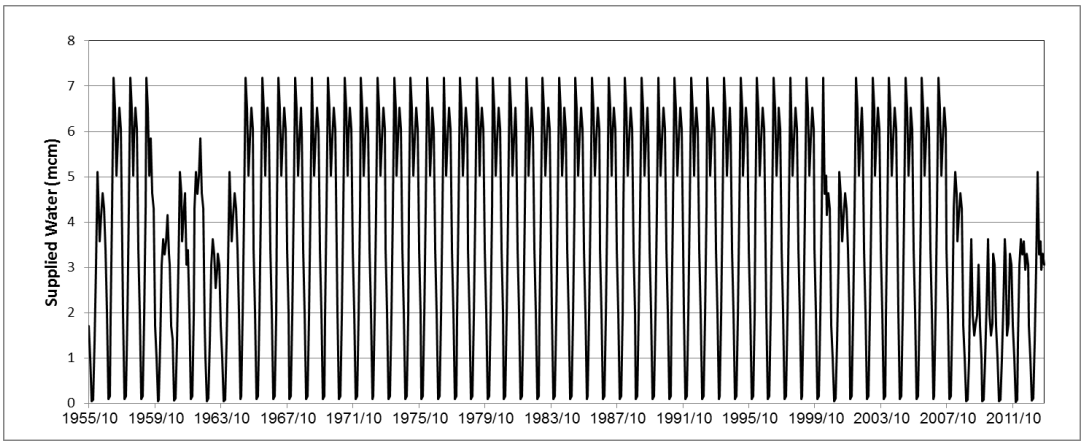


Figure 5. Optimal policy supplied water for the A5 project

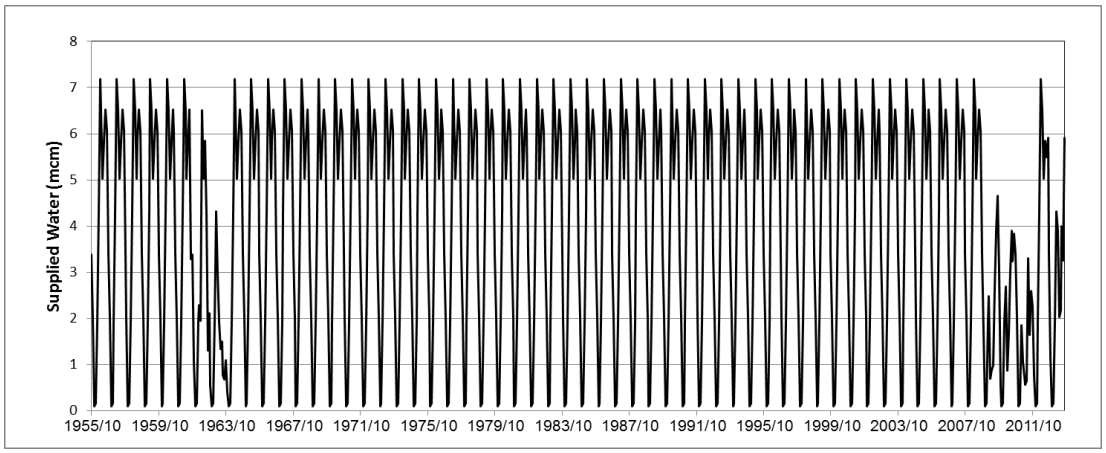


Figure 6. SOP supplied water for the A5 project

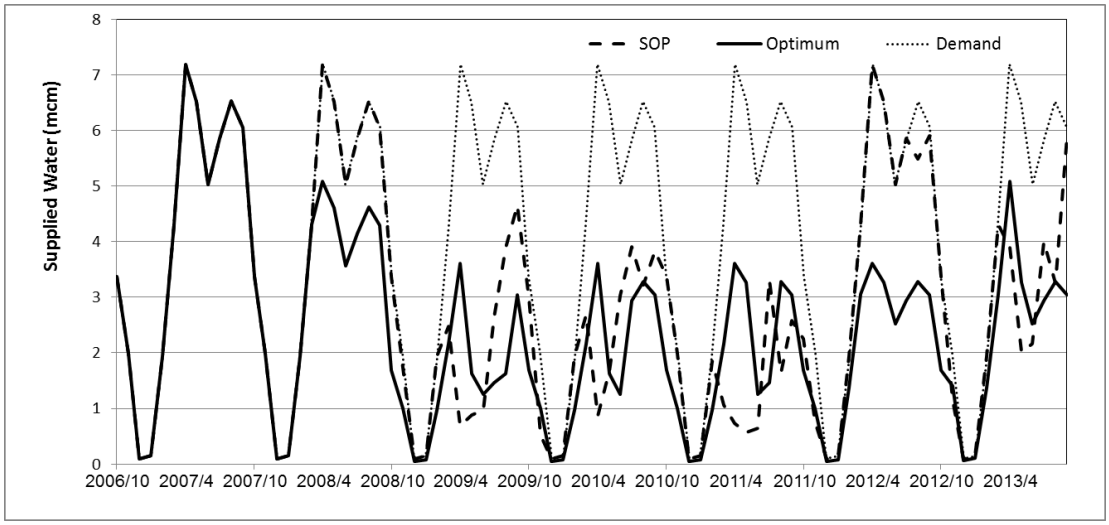


Figure 7. A5 project demand, SOP, and optimal policy supplied water in recent drought

For A5 project which is located in Zohreh system downstream (see Figure 1), the details of operation (water supplied) are presented by SOP and optimal rule-curves in Figures 5 and 6. Visual comparison of two policies' performances (SOP and optimal rule-curve with hedging) are illustrated in Figure 7. This figure graphically shows the superiority of the new routine through the smooth behavior, good correlation with demand and less severe deficits.

Since this optimal operation policy is prescribed based on the whole 58-year historical records, trigger water levels are general and synchronized for all hydrological conditions experienced including successive droughts, particularly 9-year in the initial and 7-year recent drought in the last stages of the record. But if in a particular event another policy performs better (SOP in 1999-2000, Figures 5 and 6), it corresponds to the peculiar hydrologic attributes of that event. However, it is known that in the SOP, if insufficient water is available to satisfy the demands, the reservoir releases all the water available and becomes empty; if inflow is high, the reservoir will fill and spill its excess water. Becuse this policy does not provide a mechanism for decreasing release in the early stages or during indications of impending drought.

Optimal rule-curves coupling to hedging rules for Kheirabad, Kosar, and Chamshir reservoirs are shown in Figs. 8–10. These monthly trigger levels of reservoir are optimized for management during droughts.

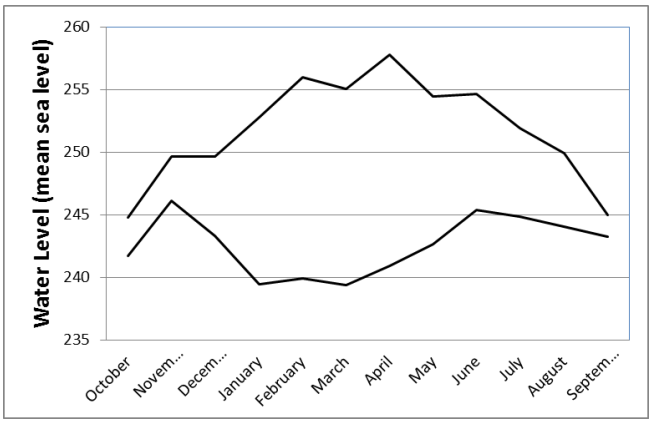


Figure 8. Optimal rule-curves for Kheirabad

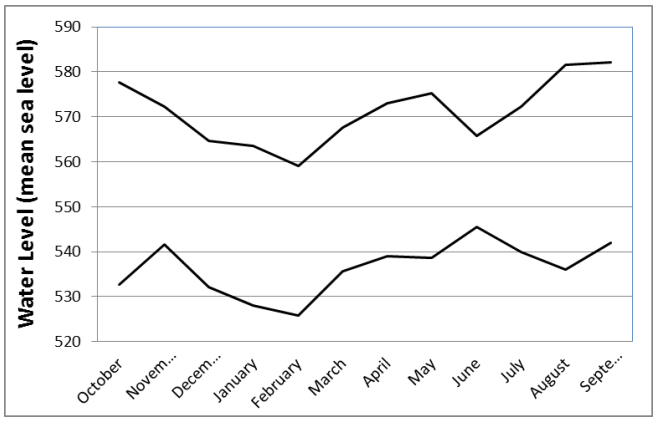


Figure 9. Optimal rule-curves for Kosar

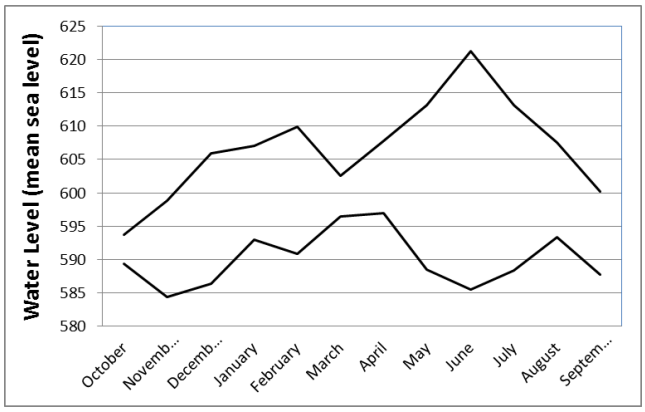


Figure 10. Optimal rule-curves for Chamshir

The rule-curves structures illustrated in Figures 8-10 indicate that the implementation of Formulas 5 and 6 in the developed model algorithm as constraints is logical. As a result, the difference between two susequent coordinate points and the difference between the first and the last month of water year is kept bounded based on the considerations and operational views of the water resource managers.

5. Conclusion

In this paper, a drought management simulation-optimization model was introduced to derive the rule-curves and rationing factors for multi-purpose multi-reservoir systems. The system of reservoirs and demands is mathematically simulated using a flexible and general WEAP model. In the simulation model, all physical constraints of the case-study are systematically satisfied. In a higher level, a self-adaptive GA is used to optimize the reservoirs rule-curves and hedging factors with the objective function of modified shortage index (MSI). The model was applied to a real three-reservoir drought-stricken system. After the optimization, the system performance was compared to the results of the SOP. The rule-curves derived from the historical 58-year period data systematically consider different climate conditions experienced by the system and would be able to manage similar conditions in the future. The proposed operating rules can mitigate the water shortage consequences by rationing current demands and saving water for future uses in drought periods. In terms of agriculture, public and hydropower supply of demands, with regard to the performance criteria, the model outperforms the SOP.

6. References

1. Tu M. Y., Hsu N. S., Yeh W. W. G., (2003). Optimization of reservoir management and operation with hedging rules. Journal of Water Resources Planning and Management, DOI: 10.1061/(ASCE)0733-9496(2003)129:2(86), 86–97.
2. Shih J. S., ReVelle C., (1994). Water-supply operations during drought: Continuous hedging rule. Journal of Water Resources Planning and Management, DOI: 10.1061/(ASCE)0733-9496(1994)120:5(613), 613–629.
3. Srinivasan K., Philipose M. C., (1996). Evaluation and selection of hedging policies using stochastic reservoir simulation. Water Resources Management, 10(3), 163–188.
4. Srinivasan K., Philipose M. C., (1998). Effect of hedging on over year reservoir performance. Water Resources Management, 12(2), 95–120.

5. Draper A. J., Lund J. R., (2004). Optimal hedging and carryover storage value. Journal of Water Resources Planning and Management, 130(1), 83– 87.
6. You J. Y., Cai X. M., (2008a). Hedging rule for reservoir operations. 1. A theoretical analysis. Water Resources Research, 44(1), W01415.
7. You J. Y., Cai X. M., (2008b). Hedging rule for reservoir operations. 2. A numerical model. Water Resources Research, 44(1), W01416.
8. Shiau J. T. (2009). Optimization of reservoir hedging rules using multi-objective genetic algorithm. Journal of Water Resources Planning and Management, DOI: 10.1061/(ASCE)0733-9496(2009)135:5(355), 355–363.
9. Shiau J. T., (2011). Analytical optimal hedging with explicit incorporation of reservoir release and carryover storage targets. Water Resources Research, VOL. 47, W01515, DOI: 10.1029/2010WR009166.
10. Eum H., Kim Y., Palmer R.N., (2011). Optimal drought management using sampling stochastic dynamic programming with a hedging rule. Journal of Water Resources Planning and Management 137(1):113–122. DOI: 10.1061/(ASCE)WR.1943-5452.0000095.
11. Dariane A., B., Karami F., (2014). Deriving Hedging Rules of Multi-Reservoir System by Online Evolving Neural Networks. Water Resources Management. DOI: 10.1007/s11269-014-0693-0.
12. Ahmadianfar I., Adib A., Taghian M., (2016). Optimization of Fuzzified Hedging Rules for Multipurpose and Multi-reservoir Systems. DOI: 10.1061/(ASCE)HE.1943-5584.0001329.
13. Huang C., Zhao J., Wang Z., Shang W., (2016). Optimal Hedging Rules for Two-Objective Reservoir Operation: Balancing Water Supply and Environmental Flow. Journal of Water Resources Planning and Management, 04016053, DOI: 10.1061/(ASCE)WR.1943-5452.0000699.
14. Shih J. S., ReVelle C., (1995). Water supply operations during drought: A discrete hedging rule. European Journal of Operational Research, 82(1), 163–175.
15. Tu M. Y., Hsu N. S., Tsai F. T. C., Yeh W. W. G., (2008). Optimization of hedging rules for reservoir operations. Journal of Water Resources Planning and Management, DOI: 10.1061/(ASCE)0733-496(2008)134:1(3), 3–13.
16. Guo X., Hu T., Zeng X., Li X., (2013). Extension of parametric rule with the hedging rule for managing multi-reservoir system during droughts. Journal of Water Resources Planning Management, 10.1061/(ASCE) WR.1943-5452.0000241, 139–148.
17. Taghian M., Rosbjerg D, Haghighi A., Madsen H., (2013). Optimization of Conventional Rule-curves Coupled with Hedging Rules for Reservoir Operation. Journal of Water Resources Planning and Management, DOI: 10.1061/(ASCE)WR.1943-5452.0000355.
18. Ashrafi S.M., Ashrafi S.F., Moazami S., (2017). Developing Self-adaptive Melody Search Algorithm for Optimal Operation of Multi-reservoir Systems. Journal of Hydraulic Structures, 3(1):35-48, DOI: 10.22055/jhs.2017.13282.
19. Zhao J., Cai X., Wang Z., (2011). Optimality conditions for a two-stage reservoir operation problem. Water Resources Research, 47, W08503, DOI: 10.1029/2010WR009971.
20. Hydrologic Engineering Center, Reservoir yield; generalized computer program, 23-J2-L245. U.S. Army Corps of Engineers, Davis, Calif, 1966.

21. Hydrologic Engineering Center, Hydrologic engineering methods for water resources development; Vol. 8, reservoir yield, U.S. Army Corps of Engineers, Davis, Calif, 1975.
22. Hsu S. K., (1995). Shortage indices for water-resources planning in Taiwan. Journal of Water Resources Planning and Management, 121(2), 119–131.
23. Ahmadi Najl A., Haghighi A., Samani H.M.V., (2016). Deriving Optimum Trade-Off between the Benefits and Costs of Inter-basin Water Transfer Projects. International Journal of Optimization in Civil Engineering, 6(2):173-185.
24. Suen J. P., Wayland E. J., (2006). Reservoir management to balance ecosystem and human needs; Incorporating the paradigm of the ecological flow regime. Water Resources Research, VOL. 42, W03417, DOI: 10.1029/2005WR004314.