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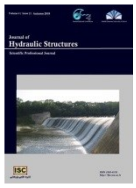

Application correlation algorithm to create a new critical depth equation for gradually varied flow in trapezoidal channel using teaching–learning and studying

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Abstract

The critical depth is a hydraulic factor of the flow, it plays a particularly important role in studying and designing for open channels, especially during identification of water surfaces and analyzing to determine the phenomenon of a hydraulic jump in open channel. In practice, when calculating the critical depth, only the rectangular and isosceles triangle channels have the theory equation, in other circumstances in calculating by semi empirical equations.

This paper presents the general method to compute the critical depth of trapezoidal channel, the case study methodology was chosen to analyze the application of existing formulas and then offering a new equation to compute the critical depth based on the optimization algorithm in MS Excel software. This new equation will help to obtain more accurate result, which relative error of the equation is less than 0.61%, this equation has a simple structure, easy to calculate with small errors to meet the conditions to quickly calculate the critical depth, this equation is also suitable for teaching–learning and studying in the field of hydraulics.

Keywords: Critical Depth, Non-Uniform Flow, Trapezoidal Chanel, Critical Flow, Specific Energy.

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1. Introduction

While studying and drawing the water-surfaces in open channel for gradually varied flow, calculating of critical depth plays an important role in the design [1-5], on the evidence of critical depth and normal depth find what kind of water-surface styles, especially hydraulics jump phenomenon. It's difficult to result exactly of critical depth of the trapezoidal channel [2][6].

The critical depth of open-channel is used to determine the type of water surfaces (one of twelve basic water surface types), it is not only applied for the calculation and determination of the hydraulic jump phenomenon, but also used in determining the state of open flow [5].

In the calculating critical depth, it was solved an equation of the sixth degree, it's difficult to find exactly result, but using trial and error method, it takes a lot of time. Determining the formula for calculating the critical depth by the method of solving the 6th equation is often for complicated mathematical formulas, as Wang and Sha (1995)[7], Zhengzhong Wang (1998)[8], Vatankhah and Easa (2011)[9], Vatankhah (2013)[10], or using experimental formulas, as Agorotskill (1930)[11], Straub (1982)[12], or expand the geometric shape of trapezoidal channels to analyze in the shape of isosceles triangular channel, as Arvanaghi, et al. (2015)[13], or using Newton's gradual method, as Cheng et al (2018)[14]. The results show that there will be the semi empirical equations, but the equations are complicated mathematical formulas, so the equations help to compute critical depth quickly and get permissible error.

There are various methods to find critical depth, each method has advantages and disadvantages. Using the knowledge of hydraulics, studying the relation between some factors, and optimization the algorithm of Microsoft Excel to create a new critical depth equation allow to obtain high accuracy results (the error is less than 0.61% - Table 4). Moreover, the method is faster in compared with other methods.

2. Theory

2.1 Baseline analysis to determine the critical depth

The flow of critical state, when Froude number is equal one. It means, the specific energy has the smallest value in a section for a given discharge [3][5][11]

+ Specific energy

$$e = y + \frac{\alpha Q^2}{2gA^2} \quad (1)$$

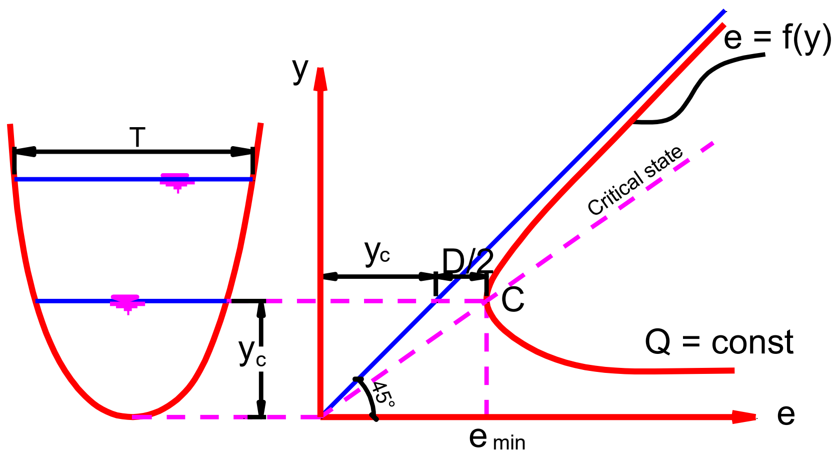


Figure 1. Specific energy graph [5][11][12]

So, function e = f(h) has graph like figure 1 [3][5][11], it is extremis of a functional where there is minimum specific energy, at that position de/dh = 0

$$\Rightarrow\ 0 = 1 - \frac{\alpha Q^2}{g A_c^{\ 3}} T_c$$

$$\Rightarrow\ \frac{\alpha Q^2}{g} = \frac{A_c^3}{T_c} \qquad (2)$$

Where

$$\frac{dA}{dh} = T_c \qquad (3)$$

From (2), y_c only dependences on Q, A. When it Q up, y_c increase and vice versa.

2.2 General critical depth method

+ Compution: $\frac{\alpha Q^2}{g}$

+ If assuming y with m, b to compute A, T and computing value $\frac{A^3}{T}$

+ From (2) comparing this value, if (2) is correct, when y = y_c.

For quick calculations and reuse, we can make a table or drawing curve, at this value point $\frac{A^3}{T} = \frac{\alpha Q^2}{g}$, we find y_c.

3. Critical depth methods of trapezoidal channel

3.1 General function

The study of trapezoidal channel (fig 2), consider that:

$$T_c = b + 2m.y_c;\ A_c = (b + m.y_c)y_c$$

From equation (2) to show that [11]:

$$\Rightarrow \frac{\alpha Q^2}{g} = \frac{A_c^3}{T_c} = \frac{(b + m.y_c)^3 y_c^3}{b + 2m.y_c} \tag{4}$$

For a given b, m, Q and α, from equation (4) to find y_c. It is an equation of the sixth degree, we cannot get root of an equation directly. Computed y_c can use trial and error method to get most accurate value, but it takes long time and is not advantage to design channel, if you use a software to calculate the need to program or use the on-line solution you need to the network.

3.2 Semi empirical equations

There are some Semi empirical equations to calculate critical depth as:
+ The formula of Agorotskill (1930, Soviet Union) [11].
Based on the empirical analysis between the critical depth of the rectangular channel and the trapezoidal channel, the equation is described as follows:

$$y_c = \left(1 - \frac{\sigma_N}{3} + 0.105\sigma_N^2\right) y_{CN} \tag{5}$$

Where:

$$\sigma_N = \frac{m y_{CN}}{b} \quad , \quad y_{CN} = \sqrt[3]{\frac{\alpha Q^2}{g b^2}}$$

+ The formula of Straub (1982) [12]:
Based on analyzing the empirical data on the critical depths of the trapezoidal channel, the equation is shown as follows:

$$y_c = 0.81\left(\frac{\Psi}{m^{0.75} b^{1.25}}\right)^{0.27} - \frac{b}{30m} \tag{6}$$

where:

$$\Psi = \frac{\alpha Q^2}{g}$$

This formula cannot use for calculating rectangular and triangular section.
+ The formula of Swamee (1993) [15].

$$y_c = \left[\left(\frac{g b^2}{Q^2}\right)^{0.7} + \left(\frac{g m^2}{2Q^2}\right)^{0.42}\right]^{-0.476} \tag{7}$$

Apply the equation to compute rectangular channel, this exponent of equation is wrong.

+ The formula of Wang and Sha (1995) [7].
Based on equation (4) to write as:

$$x\left(1+x\right)=K(1+2x)^{\frac{1}{3}} \tag{8}$$

Where:

$$x=\frac{m.y_c}{b} \qquad K=\frac{m.y_{CN}}{b} \qquad y_{CN}=\sqrt[3]{\frac{\alpha q^2}{g}}$$

It's very difficult to solve this equation.

+ The formula of Zhengzhong Wang (1998) [8].

Wang developed equation (8) to get the following equation and using interation theory, the equation was shown that:

$$y_c=\frac{b}{2m}\left(\sqrt{1+4K\left\{1+4K\left[1+4K(1+4K)^{1/5}\right]^{1/6}\right\}^{1/6}}-1\right) \tag{9}$$

Where:

$$K=\frac{\alpha m^3Q^2}{gb^5}$$

This formula cannot use for calculating rectangular and triangular section.

+ The formula of Vatankhah and Easa (2011)[9]

From equation (8) to write as:

$$t_c^6-\varepsilon_c t_c-1=0 \tag{10}$$

Where:

$$t_c=\left(2\eta_c+1\right)^{1/3} \qquad \eta_c=\frac{m.y_c}{b} \qquad \varepsilon_c=4\left(\frac{\alpha m^3Q^2}{gb^5}\right)^{1/3}$$

To get the equation:

$$y_c=0.25\frac{b}{m}\varepsilon_c(1+0.2722\varepsilon_c^{1.041})^{-0.339} \tag{11}$$

This formula cannot use for calculating rectangular and triangular section.

+ The formula of Vatankhah (2013) [10].

From equation (10) to solve by Newton-Raphson method and reported the following equation:

$$\eta_c=-\frac{1}{2}+\frac{1}{2}\left(\frac{5t_{c0}^{0.864}+1}{6t_{c0}^{0.720}-\varepsilon_c}\right)^3$$

$$t_{c0}=1+1.161\varepsilon_c\left(1+0.666\varepsilon_c^{1.041}\right)^{0.374} \tag{12}$$

$$\eta_c=\frac{m.y_c}{b}$$

This formula cannot use for calculating rectangular and triangular section.

+ The formula of Arvanaghi et al. (2015) [13]
From equation (4) to get:

$$\lambda = \frac{m.y_c}{b} \qquad K = \frac{\alpha Q^2 m^3}{gb^5}$$

Writing (4) as:

$$\lambda^6 + 3\lambda^5 + 3\lambda^4 + \lambda^3 - 2\lambda K - K = 0$$

Root of an equation

$$\lambda = -1.55K^{0.06} + 1.68K^{0.18} + 0.644 \tag{13}$$

The main limitation of this method is $1 < b/y_c < 6$, so a range of caculation has been scaled down, this erorr is less than 1.22% [13]

+ The formula of Tiejie Cheng et al (2018) [14]
Using the theory of algebraic inequality and Trial and error method to find critical depth as:

$$y_{c,s+1} = \sqrt[3]{\frac{\alpha Q^2\left[(m_1+m_2)y_{c,s}+b\right]}{g.\cos\theta} \Bigg/ \left[\frac{(m_1+m_2)y_{c,s}}{2}+b\right]} \tag{14}$$

If $m_1 = m_2 = 0$ this equation computes for rectangular channel, or $m_1 = m_2 = 0$ and $b = 0$, this equation is right for triangular section. This equation has true value after computing four or more miscarriages in a row.

+ The Equation is given by Farzin Salmasi (2020) [16]
Using artificial neural network (ANN) techniques to find critical depth as:

$$y_c = 0.856\left(\frac{Q}{b}\right)^{0.473}\left(\frac{1}{g^{1/3}m^{0.291}}\right)^{0.242} - 0.083\left(\frac{b}{m}\right) \tag{15a}$$

Or

$$y_c = -0.048\left(\frac{m.Q^{4/3}}{b^{7/3}g^{2/3}}\right) + 0.796\left(\frac{m.Q^{2/3}}{b^{5/3}g^{1/3}}\right) \tag{15b}$$

The formula of Farzin Salmasi (2020) has used ANN to analyze the data series and find the equation of critical depth. Maximum errors of this equation are 0.998 and 0.996 respectively [16].

Realizing: Add a dose of Trial and error method to compute y_c to get the most accurate value, but it takes a lot of time and uses sophisticated algorithms, if use graphics to find y_c, it has error from both subjective and objective reasons. Thus, applying the semi empirical equations to get the best results, but most of the equations are not exactly with m = 0 or get accurate results if m

= 0 but it has big error when m = 0.
So, this study focused on analyzing the relationships of hydraulics factors on trapezoidal channel. In combination with correlation analysis algorithm of Microsoft Excel to find out a new equation, which it gets the result correctly and quickly.

4. Setting up a critical depth equation of trapezoidal channel

4.1 Theoretical analysis and proposed formula

With

$$k = \frac{m.y_c}{b} \Rightarrow m.y_c = b.k$$

From (4):

$$\frac{\alpha Q^2}{g} = \frac{A_c^3}{B} = \frac{(b + b.k)^3}{b + 2b.k} y_c^3 = \frac{(1+k)^3}{1+2k} b^2 y_c^3$$

$$\Rightarrow \frac{\alpha Q^2}{gb^2} = \frac{(1+k)^3}{1+2k} y_c^3 \quad (16)$$

+ Computing critical depth for a rectangular channel (y_{CN}) by bed width (b) [3][5][11]:

$$y_{CN} = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} \quad (17)$$

From (16):

$$y_{CN}^3 = \frac{(1+k)^3}{1+2k} y_c^3$$

$$\Rightarrow \frac{y_{CN}}{y_c} = \frac{1+k}{\sqrt[3]{1+2k}} \quad (18)$$

Equation (18) uses to compute critical depth of trapezoidal channel.

4.2 Study method

Consider a trapezoidal channel, a given discharge (Q), coefficient of slope (m) and bed width of trapezoidal channel (b), solution:

+ Computing critical depth in a trapezoidal channel (y_c) by trial and error method from equation (4). It is the method that obtains the highest accurate results.
Flowing (18), given as:

$$g(k) = \frac{1+k}{\sqrt[3]{1+2k}}$$

Table 1. Computation result of geometric coefficients

Q (m^3/s)	b (m)	m	y_c from trial and error method (m)	y_{CN} (m)	k	g(k)
1	1	1	0.405	0.467	0.405	1.153
1	2	1	0.280	0.294	0.140	1.051
1	3	0.5	0.222	0.225	0.037	1.012
2	1	1.25	0.580	0.742	0.725	1.279
2	2	1.5	0.418	0.467	0.314	1.118
2	3	2	0.330	0.356	0.220	1.080
5	1	1	0.988	1.366	0.988	1.383
5	2	1	0.754	0.860	0.377	1.141
5	3	0.5	0.633	0.657	0.106	1.037
10	1	1.25	1.328	2.168	1.660	1.633
10	2	1.5	1.048	1.366	0.786	1.303
10	3	2	0.856	1.042	0.571	1.218
20	2	1	1.645	2.168	0.823	1.318
20	3	0.5	1.514	1.655	0.252	1.093
50	5	1.25	1.845	2.168	0.461	1.175
50	2	1.5	2.380	3.994	1.785	1.678
50	3	2	2.011	3.048	1.341	1.516
50	4	1.25	2.024	2.516	0.633	1.243
60	3	1.5	2.364	3.442	1.182	1.456
60	5	2	1.895	2.448	0.758	1.292
70	5	1	2.309	2.713	0.462	1.175
80	4	1	2.727	3.442	0.682	1.262
100	10	0.5	2.092	2.168	0.105	1.036

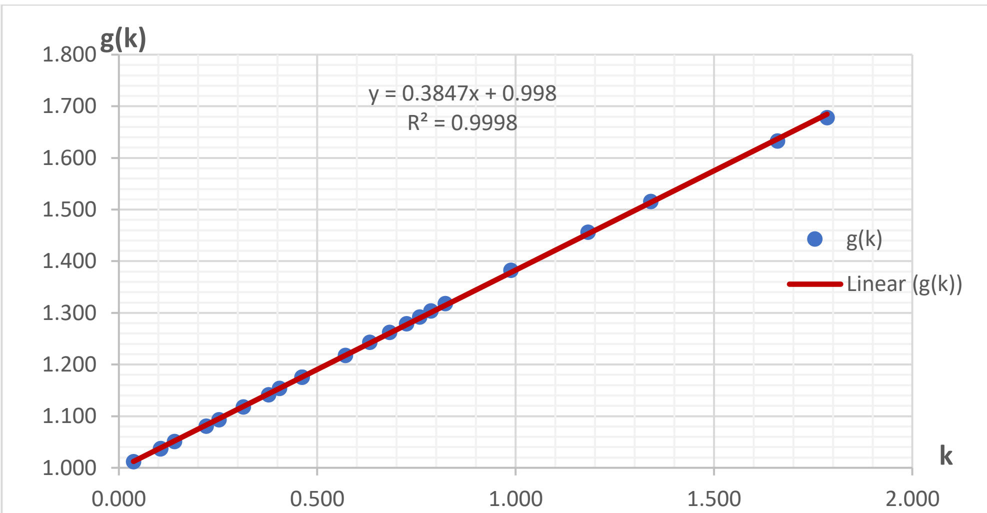


Figure 2.Correlogram between k and g(k)

As a result of Fig 2, the equations will be described as below:

$$g\left(k\right) \approx 0.384k \; + \; 1 \tag{19}$$

From (18) and (19), the equation is defined as follows:

$$\frac{y_{CN}}{y_c} = \frac{5m}{13b}.y_c + 1$$
$$\frac{5m}{13b}.y_c^2 + y_c - y_{CN} = 0 \tag{20}$$

The equation (20) is a quadratic equation with a unknowns (y_c), if caculating for rectangular (m = 0), a critical depth of trapezoidal channel becomes a critical rectangular depth ($y_c = y_{CN}$), when m ≠ 0, this equation can sovle to get a root:

$$y_c = \frac{13.b}{10.m}\left(\sqrt{1 + \frac{20m}{13b}.y_{CN}} - 1\right) \tag{21}$$

Where: y_{CN} caculates from Eq. (17)
Equation (21) uses to calculate a critical depth of trapezoidal channel.
The data analysis is about 1000 critical depths, also obtained the same results as Fig 2. Therefore, formula (21) is very suitable in calculating the critical depth for trapezoid channel. Survey data was shown in Table 2.

Table 2. Variable range of parameters

Parameters	Min	Max
Q (m^3/s)	1	120
b (m)	1	10
m	0.5	2
y_c (m)	0.14	3.9
k	0.1	2.5

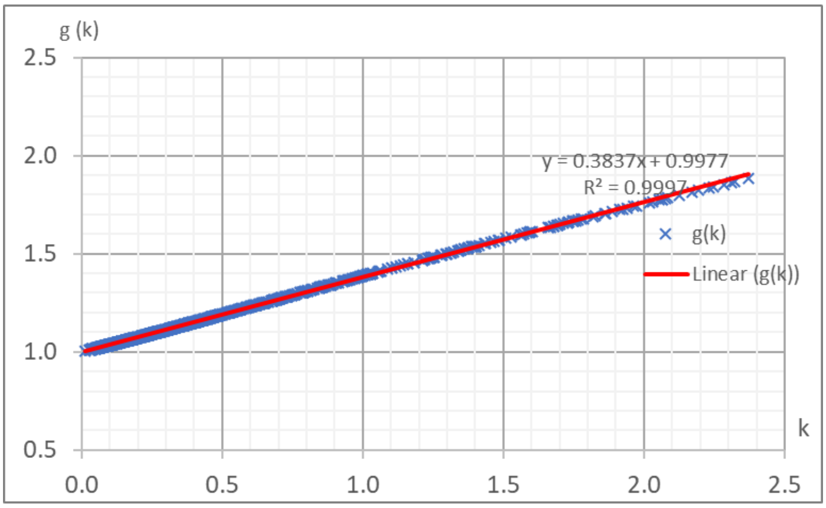


Figure 3. Relationship between k and g (k) as a linear function

Comparison between Fig.2 and Fig.3 show that correlation function between g (k) and k does not change much, the coefficient R^2 is approximately the same, so the equation (21) ensures the calculation of the critical depth.

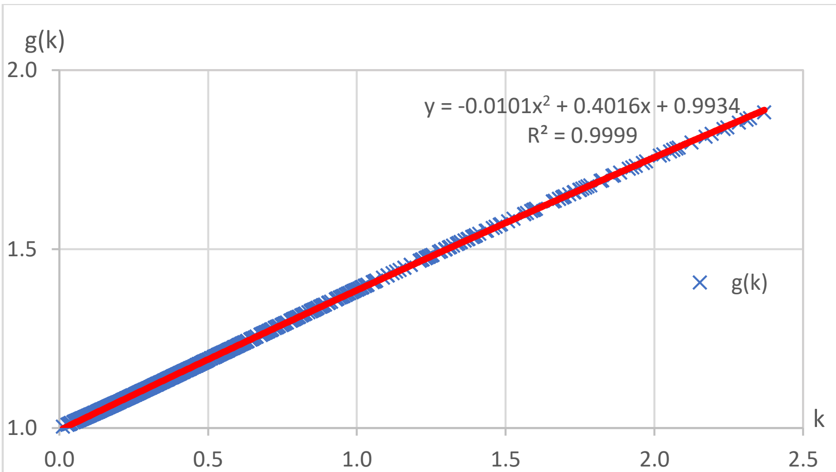


Figure 4. Relationship between k and g (k) in quadratic function

$$\frac{0.0101m^2}{b^2}y_c^3 - \frac{0,4016m}{b}y_c^2 - 0,9934y_c + y_{CN} = 0 \quad (22)$$

A solution of the equation (22) is the critical depth of trapezoidal channel, it is found as follows:

With assuming:

$$A=\frac{0.0101m^2}{b^2} \qquad B=-\frac{0.4016m}{b}$$

$$C=-0.9934 \qquad D=y_{CN}$$

$$\Delta=B^2-3.A.C \qquad k=\frac{9.A.B.C-2.B^3-27.A^2.D}{2\sqrt{\Delta}}$$

The fomula of critical depth writes as:

$$y_c=\frac{2\sqrt{\Delta}.\cos\left(\frac{\arccos(k)}{3}-\frac{2\pi}{3}\right)-B}{3A} \tag{23}$$

In equation (23), it is necessary to get the Pi of at least 4 decimal places ($\pi \approx 3.1416$). If calculating on excel, the results are quite accurate and the maximum error is about 0.3%. If the calculation is rounded to 3 decimal places (A, B, C, D, Δ, k, arccos(k) and so on), the margin of error in calculating critical depth is very big.

5. Calculation and assessment of critical depth equation in trapezoidal channel

+ Applying equation (21) to compute critical depth of trapezoidal channel (table 3).
+ Test results based on error evaluation between the semi-empirical formulas and the Trial and error method from equation (4)

Equation error of y_c to use "Mean absolute percentage error" (MAPE):

$$MAPE=\frac{\left|y_{c\ (Eq21)}-y_{c\ (Eq4)}\right|}{y_{c\ (Eq4)}}\times 100\ (\%) \tag{24}$$

Where

ε_{error}: error between equation proposed and accurate result (%)

$y_{c\ Eq21}$: computed value by equation proposed

$y_{c\ Eq4}$: computed value by the Trial and error method from equation (4)

Based on value of the trial and error method from equation (4), comparative analysis between data series by Nash index [17].

$$Nash=1-\frac{\sum\left(y_{c\ Eq21}-y_{c\ Eq4}\right)^2}{\sum\left(y_{c\ Eq21}-y_{c\ ave}\right)^2} \tag{25}$$

Where: $y_{c\ ave}$: Average value of data series by computing equation (4)

Evaluation Criteria: Nash is to one, the result of equation proposed will be correct.

Table 3. Evaluation of test results by Nash

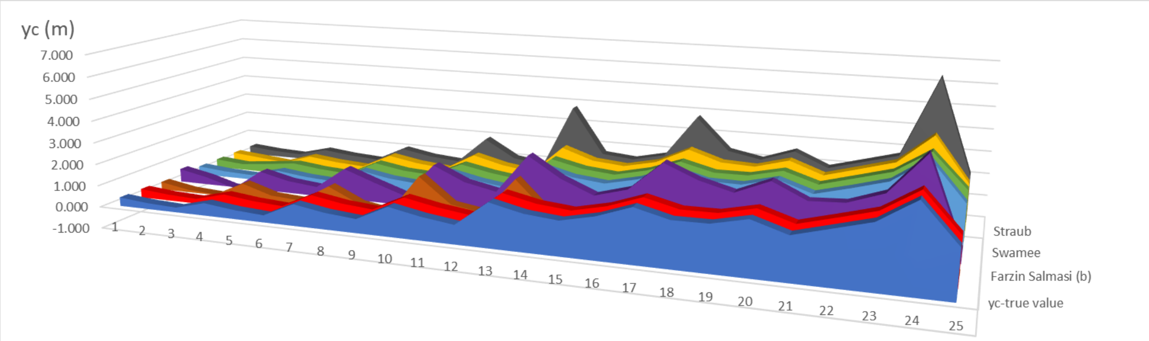
Critical	Equation (21)
Nash	0.999

Remarks: From the results of the analysis, the value of y_c in equation (21) gives relatively good results, calculating y_c under different flow conditions, showing the error of less than 0.61% and coefficient Nash is very close to 1.

Table 4. Computing y_c by equation proposals

No	Q (m^3/s)	b (m)	m	$y_{c\ Eq4}$ from trial and error method (4) (m)	y_{CN} (m)	$y_{c\ Eq21}$ form Eq (21) (m)	ε_{error} (%)
1	1	1	1	0.405	0.467	0.404	0.25
2	1	2	1	0.280	0.294	0.279	0.36
3	1	3	0.5	0.222	0.225	0.222	0.00
4	2	1	1.25	0.580	0.742	0.58	0.00
5	2	2	1.5	0.418	0.467	0.417	0.24
6	2	3	2	0.330	0.356	0.328	0.61
7	5	1	1	0.988	1.366	0.989	0.10
8	5	2	1	0.754	0.86	0.751	0.40
9	5	3	0.5	0.633	0.657	0.631	0.32
10	10	1	1.25	1.328	2.168	1.325	0.23
11	10	2	1.5	1.048	1.366	1.049	0.10
12	10	3	2	0.856	1.042	0.855	0.12
13	20	1	1	1.973	3.442	1.962	0.56
14	20	2	1	1.645	2.168	1.647	0.12
15	20	3	0.5	1.514	1.655	1.509	0.33
16	50	5	1.25	1.845	2.168	1.842	0.16
17	50	2	1.5	2.380	3.994	2.372	0.34
18	50	3	2	2.011	3.048	2.011	0.00
19	50	4	1.25	2.024	2.516	2.024	0.00
20	60	3	1.5	2.364	3.442	2.366	0.08
21	60	5	2	1.895	2.448	1.895	0.00
22	70	5	1	2.309	2.713	2.304	0.22
23	80	4	1	2.727	3.442	2.727	0.00
24	100	2	1	3.719	6.34	3.703	0.43
25	100	10	0.5	2.092	2.168	2.084	0.38

Analyzing the critical depth by some equations, the calculation results are shown in Figure 5.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
yc-true value	0.405	0.280	0.222	0.580	0.418	0.330	0.988	0.754	0.633	1.328	1.048	0.856	1.973	1.645	1.514	1.845	2.380	2.011	2.024	2.364	1.895	2.309	2.727	3.719	2.092
Eq 21	0.404	0.279	0.222	0.58	0.417	0.328	0.989	0.751	0.631	1.325	1.049	0.855	1.962	1.647	1.509	1.842	2.372	2.011	2.024	2.366	1.895	2.304	2.727	3.703	2.084
Farzin Salmasi (b)	0.361	0.115	0.029	0.705	0.271	0.185	0.998	0.325	0.084	1.875	0.748	0.518	2.171	0.750	0.198	0.375	1.810	1.320	0.531	1.086	0.664	0.361	0.543	1.559	0.075
Farzin Salmasi (a)	0.629	0.347	-0.053	0.906	0.581	0.435	1.441	0.932	0.454	2.016	1.371	1.074	2.854	1.950	1.336	1.751	3.061	2.441	2.049	2.688	1.989	2.066	2.605	4.364	0.562
Swamee	0.399	0.276	0.22	0.576	0.411	0.324	0.989	0.741	0.623	1.345	1.043	0.846	2.004	1.639	1.486	1.818	2.413	2.026	2.005	2.374	1.883	2.275	2.704	3.773	2.058
Arvanaghi,	0.406	0.286	0.426	0.582	0.419	0.331	0.993	0.755	0.671	1.334	1.052	0.859	1.983	1.652	1.516	1.851	2.392	2.02	2.032	2.374	1.903	2.316	2.738	3.737	2.221
Straub	0.404	0.279	0.147	0.581	0.419	0.331	1.009	0.759	0.628	1.422	1.061	0.859	2.171	1.678	1.551	1.874	2.591	2.119	2.058	2.47	1.931	2.352	2.785	4.094	2.114
Agorotskin	0.405	0.281	0.222	0.579	0.419	0.33	1.012	0.754	0.634	1.882	1.05	0.854	3.774	1.652	1.516	1.843	3.769	2.305	2.02	2.538	1.896	2.307	2.722	6.33	2.093

Figure 5. Graphic of predictive formulas and exact value

From the calculation results, it shows that the formulas of Wang (1998), Vatankhah, Easa (2011) and Vatankhah (2013) were the smallest error, if it was compared with real results, it was 0.06% (case: Q = 20 m^3/s, b = 2m and m = 1), 0,1% and 0.05 (case: Q = 110 m^3/s, b = 10m and m = 0.5) respectively. The formula of Farzin Salmasi (a, b2020) also has negative values and the largest error is up to 124% (case: Q = 1 m3 / s, b = 3m and m = 0.5), the calculation results are unstable and large error fluctuations.

Other formulas have a large error at some spikes, as the equation of Agorotskill (1930) has the maximum error up to 91.28% (case: Q = 20 m3 / s, b = 1m and m = 1) and many other points have relatively large error, maximum error of Straub (1982)'s equation, H. Arvanaghi et al (2015)'s equation are 33.87% (case: Q = 1 m^3/s, b = 3m and m = 0.5) and (Case: Q = 5 m^3/s, b = 3m, m = 0.5 and Q = 100 m^3/s, b = 10m, m = 0.5) respectively. In other cases of calculation, there are small stability errors and no unusual large values. The maximum error of Swamee (1993)'s equation is 1.85% (case: Q = 20 m^3/s, b = 3m and m = 0.5) and the error of Tiejie Cheng et al (2018)'s equation is equivalent to the proposed equation (21) which is 0.61%. The formulas with large mutation errors only occur in a few cases in the calculation result series, while the majority cases are quite stable and have errors small enough to accept calculation results.

The maximum error of the proposed formula (21) is quite small (0.61%) and other cases have much smaller error, in addition to the simple formula and quick calculation. So this formula is very suitable in researching and teaching when it is necessary to have fast data and small error.

Other formulations are also suitable for each case study and have many scientific meanings in hydraulic engineering.

6. Conclusions and Suggestions

While studying unsteady flow in open channel, the most important factor for identification water surface is the critical depth. For the trapezoidal channel, the semi empirical equation was often used to calculate the critical depth, but the closer a side slope coefficient (m) is to zero, the calculation error of the empirical equation will become larger, because the changing law of the critical depth from a non-linear function to a linear function.
Based on analytical theory and applying a correlation algorithm in Microsoft Excel, it has found two formulas to calculate the critical depths, which are equations (21) and (23). However, equation (23) calculates complicated, if rounding the calculated values, it has large error (over 10%), so the equation (21) is proposed to apply for more extensive calculation with small error (under 0.61%).
Meanwhile, the equation (21) helps to compute critical depth quickly, it may be made water surface quickly and advantageously.

The report examines the causes of critical depth and applying correlation analysis of Microsoft Excel with linear and non-linear equation, that correlation coefficients is large number (near one), this analysis prove fruitful.

The correlation algorithm has a lot of strong points to find the approximate root for higher degree non-linear equation. In studying and learning, it has to find out and develop this algorithm for other field of studies.

Notation

The units shown below are SI (international system of units).

b : bed width of trapezoidal channel, m.
m : Side slope of the channel
yc : critical depth of trapezoidal channel, m.
y_{CN} : critical depth of rectangular channel, m.
A : Flow area of water, m^2.
T : Top width of flow, m.
Q : Discharge, m^3/s.

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