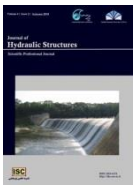
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# Numerical investigation of simultaneous effect of end sills and roughness on flow characteristics in V-shaped stepped spillways

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## Abstract

The special configuration of V- shape stepped spillways increases energy dissipation and aeration compared to the smooth spillways due to the creation of many vortices near the steps. In this research, the energy dissipation in different types of stepped spillways with various horizontal face angles has been investigated using numerical modeling. The FLUENT, was used to model the flow over V-shape stepped spillway. The k-ε realizable turbulence model was selected to model the turbulent flow. The numerical results were compared with the available experimental data. The results showed a reasonable agreement between two sets of data. Then the effects of horizontal face angle, roughness and the efficiency of the end sill were investigated by numerical modeling. According to the model results, as the horizontal face angle increased, the energy dissipation also increased. Furthermore, the efficiency of end sill on the stepped spillways increased the rate of energy dissipation about 2.8 to 3.99 percent because the end sill acted like a stilling basin. Moreover, the energy dissipation increased slightly about 0.9 to 1.94 percent by increasing the roughness. Also, areas of steps under the negative pressure that could create cavitation were determined to define the minimum negative pressure and its location in all of models. Finally, the simultaneous effect of several parameters was considered to increase the energy dissipation and the minimum negative pressure.

**Keywords:** Stepped spillway, Energy dissipation, Numerical method, Fluent software.

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## 1. Introduction

Stepped spillways with the Roller Compacted Concrete (RCC) construction procedure have achieved reputation in the recent two decades, through economic efficiency, fast construction and facility of performance. Steps operate like a large roughness for energy dissipation on stepped spillways. The stepped spillway's energy dissipation characteristics are comparable to

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those of smooth spillways with a stilling basin at the downstream [1].
There is complexity in stepped spillway hydraulics because different types of flow regimes are formed on this type of spillways, which comprised free-surface aeration, flow turbulence, and exchange among dragged air and turbulence [2]. Researches have demonstrated three kinds of flow regimes like napped, transition and skimming flows on stepped spillways. In napped flow regime, flow slid down the steps and strikes the step down like falling jet that happens in low discharge values [3]. In skimming flow, flow skims on the steps and vortices become visible in triangular area that arises in high discharge values. Transition flow regime is an impermanent state among napped and skimming flow regimes. It should be mentioned that for practical engineering purposes, skimming flows are more **appropriate** than napped flows [4].
Various researches have been accomplished on flow attributes over stepped spillways. Christodoulou [5] represented the rate of energy dissipation on stepped spillways that the ratio of the critical depth of flow to the step height is important parameter over energy dissipation. Chen et al. [6] expressed the influence of step height on energy dissipation rate is very little in the high unit discharges. Chinnarasri and Wongwises [7] examined energy dissipation in different types of stepped spillways and concluded the steps with end sills were the most efficient among other types. Gonzalez et al. [8] also researched the effect of step roughness in stepped spillways. Morovati et al. [9] and Abbasi and Kamanbedast [10] showed that decreasing the number of the steps increased energy dissipation. As well as, Barani et al. [11] recognized stepped spillways with inclined steps is more effective than stepped spillways with plain and end sill steps in increasing energy dissipation. Bai et al. [12] examined stepped spillway with various horizontal face angles and presented flow properties on the stepped spillway.
The pressure distribution on the stepped spillway has an important role because the negative pressure on the step surface can create cavitation. Many researchers have studied the pressure distribution on the step surface of stepped spillways using physical models [13, 14 and 15]. Zhang et al. [16] concluded the pressure is always negative on the top of vertical step surfaces of stepped spillway and remains negative along the entire vertical surface. In the mentioned researches, the simultaneous effect of several parameters to improve the energy dissipation in the stepped spillways was not investigated which is considered in the present research.
Recently, numerical modeling has been widely applied to model stepped spillways, especially to predict energy dissipation. In this study, FLUENT [17] has been employed to simulate the flow over the V-shaped stepped spillway and also, the k-ε realizable turbulence model has been selected to model the turbulent flow. In the present research, the simultaneous effects of important parameters on the flow properties of the V- shaped stepped spillway are investigated. For this purpose, end sills on the steps and increasing the roughness are considered and the effects of these parameters are investigated on the increasing energy dissipation and also, the location of areas with minimum negative pressure.

## 2. The spillway geometry and properties

The physical model has been constructed by Bai et al. [18] at the State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu. This model includes a water tank, a press slope portion, a flat section, a stepped spillway, a tail water section, a measuring weir and a reservoir (Figure 1). The model material is organic glass (a fully synthetic plastic material available in a vitreous state) with roughness value of 0.0085. In this paper, the roughness effect has also been investigated and the other roughness value (n= 0.012) has been applied in numerical modeling. The inlet height of the model is 12 cm, the width is 40 cm, and the outlet height of the pressed slop portion is 8 cm. The model consists of 56 steps, 28 steps are

transitional steps at the beginning of the stepped spillway, which slowly change to V-shaped and the rest of the steps have the same size and horizontal face angle. In the experimental model of Bai et al. (2018), the velocity was measured by a propeller-type current meter and pitot tube. The unit discharge was controlled by the measuring weir, and depth was also obtained by the unit discharge and the velocity. The numerical model in the present study includes the 28 final steps with five different horizontal face angles (θ=-30°, -15°, 0°, 15° and 30°). Moreover, an end sill of 0.1 m in height and 0.1 m in length has been added to the model to investigate the effect of end sill on the flow parameters. In this paper, V-shaped, inverted V-shaped, and traditional stepped spillways mean spillways with horizontal face angle greater than zero, less than zero, and equal zero, respectively. The model geometry and the horizontal face angle are shown in Figures 2 and 3 [18].

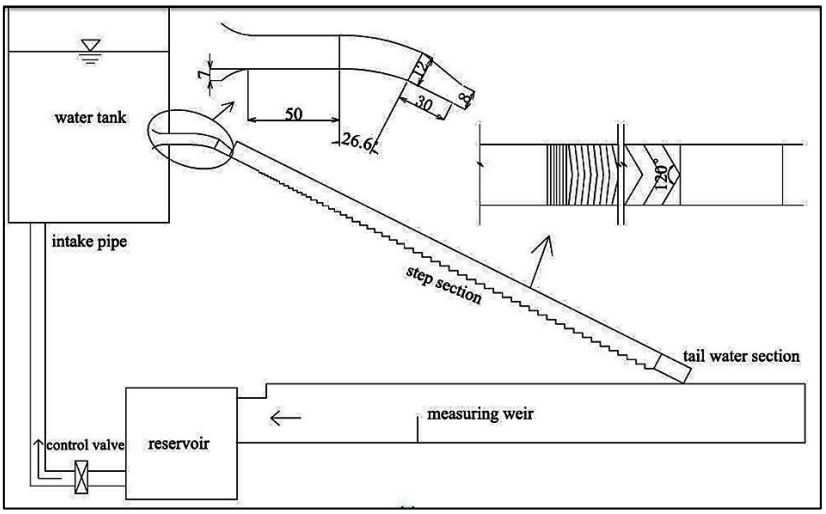



**Figure 1. Experimental set-up of Bai et al. [18]**

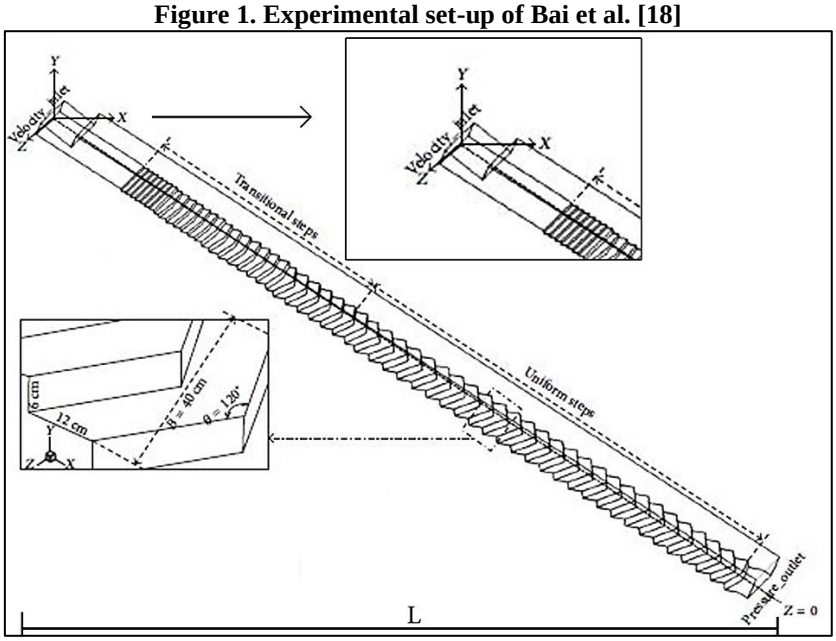
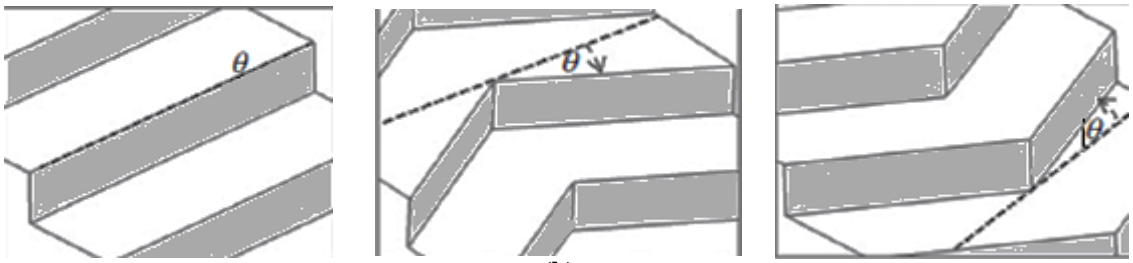



**Figure 2. The sketch of the present V-shaped stepped spillway [18]**




**(a)** **(b)** **(c)**

**Figure 3. The horizontal face angle in the model a) θ=0° b) θ=30° c) θ=-30°**

## 3. Numerical model implementation

### 3. 1. Mesh generation

Gambit software is a mesh generator preprocessor that is compatible with Fluent. This software has been utilized to design the geometric shape of the model as well as to determine the boundary conditions. When geometries of computational domain are complicated, a triangular mesh can be used in order to adjust the triangles to the boundaries. That is the reason to select the triangular mesh in the present study. The selected mesh is an unstructured triangular mesh that the cells are arranged in an optional method. Unstructured meshes with hexahedral cells were generated in this research using a tri-pave element. These grids all have a cell skewness less than 0.07. The mesh density should be high sufficient to overcome all related to flow characteristics. The mesh density is increased next to the wall and on the surface of the spillway to enhance the accuracy of the results. Figure 4 shows the mesh created for the spillway in the present study. The number of meshes generated for each step on the horizontal and vertical surfaces is 624 and 388, respectively.

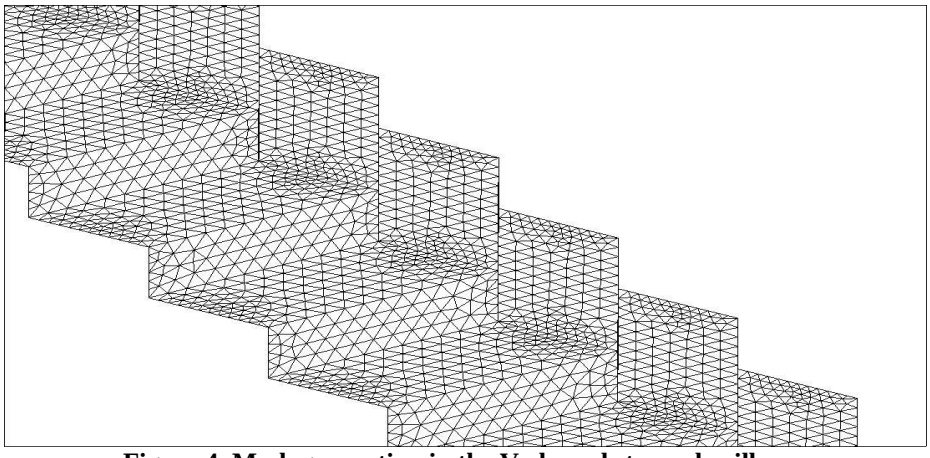


**Figure 4. Mesh generation in the V-shaped stepped spillway**

### 3. 2. Boundary conditions

The boundary conditions are inflicted to the model as follows;

The inlet boundary was defined as an inlet-velocity condition, which flow velocity is related to the unit discharges (q). For the downstream boundary, a pressure-outlet was used where the normal gradient of the pressure is set to zero. The wall was treated as no-slip velocity condition, and the near-wall regions were analyzed applying the standard wall function. For the upper

boundary condition of the stepped spillway, the free surface was defined as an inlet-pressure boundary with a pressure value of zero.

## 3. 3. Sensitivity analysis

One of the most important issues in numerical modeling is independence of the model results from the mesh size. For this purpose, grid density is changed in the model and after examining the accuracy of different grid densities, the appropriate mesh is selected. In this study, three grid types (0.015× 0.015$m^2$, 0.01×0.01$m^2$ and 0.009×0.009$m^2$) have been utilized. The accuracy of each grid type has been evaluated using Root Mean Square Error (RMSE):

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N}(y_{pi} - y_{ni})^2\right]^{0.5} \quad (1)$$

where, N is the number of data, $y_{pi}$ represents the numerical model values, and $y_{ni}$ represents the experimental model value. The results have been presented in Table 1. According to the results, the grid type II has less error than type III. The run time of grid type II is more than grid type III however, there is not any difference in the accuracy of these two grids. Therefore, the grid type II has been selected as the selected mesh for the rest of simulations. In addition, the mean relative error ($E_{rel}$) was calculated and presented in Table 1 to evaluate the accuracy of results as follows:

$$E_{rel} = \frac{\sum_{i=1}^{N}|y_{pi} - y_{ni}|}{\sum_{i=1}^{N} y_{pi}} \times 100 \quad (2)$$

**Table 1. The grid properties and corresponding errors**

| Grid type | Grid size (m) | Mesh number | RMSE (m) | $E_{rel}$ ($percent$) |
|---|---|---|---|---|
| I | 0.009×0.009 | 244651 | 0.0066 | 2.19 |
| II | 0.01×0.01 | 156519 | 0.0067 | 2.23 |
| III | 0.015× 0.015 | 92631 | 0.0102 | 3.45 |

## 3. 4. Governing equations

The governing equations consist of the continuity and Navier–Stokes equations which are indicated in equations 3, 4 for an incompressible fluid, respectively.

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (3)$$

$$\rho\left[\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j}\right] = \mu\frac{\partial^2 u}{\partial x_j \partial x_j} - \frac{\partial P}{\partial x_i} + X_i \quad (4)$$

Where ρ is the fluid density, $u_i$ refers to the velocity components in the corresponding $x_i$ Cartesian coordinates ($i = 1, 2, 3$), $t$ is the time, $P$ is the pressure, $\mu$ is the dynamic viscosity, and the $X_i$ represents a body force exerting on a unit volume [19].

The Volume of Fluid (VOF) method is widely used to track the water surface. This model was proposed by Hirt and Nichols [20]. In the VOF method, each phase is a variable that is named a volume fraction of the phase, and calculates for how much percentage of each phase exists in each cell. The sum of the volume fractions of air and water is unity in each computational cell [21].

$\alpha_w + \alpha_a = 1; \quad 0 \le \alpha_w, \alpha_a \le 1$ (5)

In this study, fluent software is applied to simulate the flow on a stepped spillway. The numerical simulation is managed using the pressure-velocity coupling algorithm that the pressure-implicit with splitting of operators (PISO) is applied. The PRESTO scheme is also used for the discretization of pressure term. The first order upwind scheme has been applied for discretization of the spatial terms in momentum and turbulence equations. Moreover, the modified HRIC scheme is used for the volume fraction equations.

In the present study, k-ε realizable turbulence model has been used to simulate the turbulent flow. Equations 6 and 7 represent turbulent kinetic energy ($k$) and its dissipation rate ($\varepsilon$) as follows [18]:

$k$ equation:

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial}{\partial x_j}(\rho k u_j) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + G_k + G_b - \rho\varepsilon - Y_m + S_k \quad (6)$$

$\varepsilon$ equation:

$$\frac{\partial(\rho\varepsilon)}{\partial t} + \frac{\partial}{\partial x_j}(\rho\varepsilon u_j) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + \rho C_1 S_\varepsilon - \rho C_2 \rho \frac{\varepsilon^2}{k + \sqrt{\nu\varepsilon}} + C_{1\varepsilon}\frac{\varepsilon}{k} C_{3\varepsilon} G_b + S_\varepsilon$$

$$C_1 = \max\left[0.43.\frac{\eta}{\eta + 5}\right]$$

$$\mu = \frac{sk}{\varepsilon}$$

$$S = \sqrt{2 S_{ij} S_{ij}} \quad (7)$$

$$S_{ij} = 0.5(\partial u_j/\partial x_i + \partial u_i/\partial x_j)$$

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon}$$

$$G_k = \mu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j}$$

Where $v$ is the turbulent kinematic viscosity; $C_{1\varepsilon}$ = 1.44, $C_2$= 1.9, $\sigma_k$= 1.0, and $\sigma_\varepsilon$ = 1.2 are the empirical constants. The equations are solved by using fluent software, and the computations are continued until a correlation is reached. The residual relative error has been selected for the correlation criterion.

## 4. Results and discussion

### 4. 1. Comparison of numerical results with experimental data

The numerical results of pressure values comparing to the experimental data of Bai et al. [12] is shown in Figure 6 for the 43rd step of V-shaped stepped spillway with θ = 30 ° for different points on horizontal surface which is shown in Figure 5. Where $B$ is the model width and $Z$ refers to the coordinate value in Z direction.

According to the Figure 6, a good agreement between the present model results and the experimental data is observed. The mean relative error of the numerical pressure results for the 43rd step of V-shaped stepped spillway on horizontal surface for $Z/B = 0$, $Z/B = 0.25$ and $Z/B = 0.5$ is about 7.5%.

The present model results indicate that it is possible to rely on the numerical model and simulate more models and compare the results to achieve a stepped spillway with more energy dissipation.

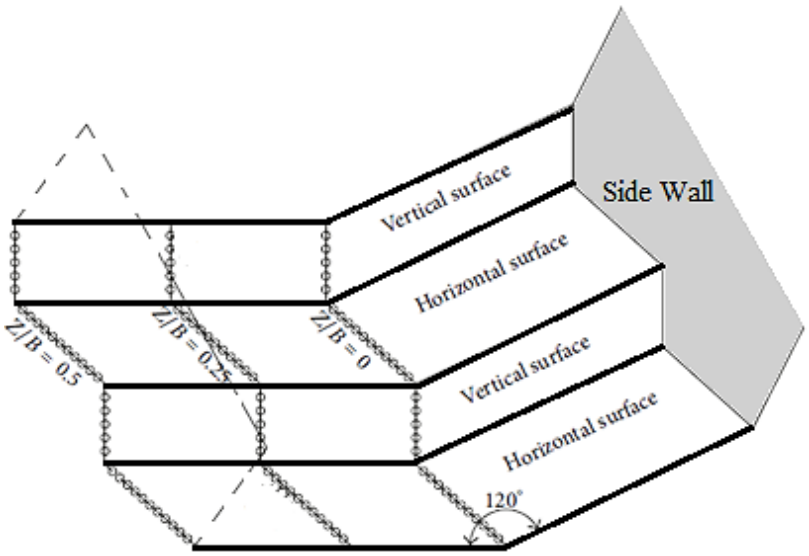
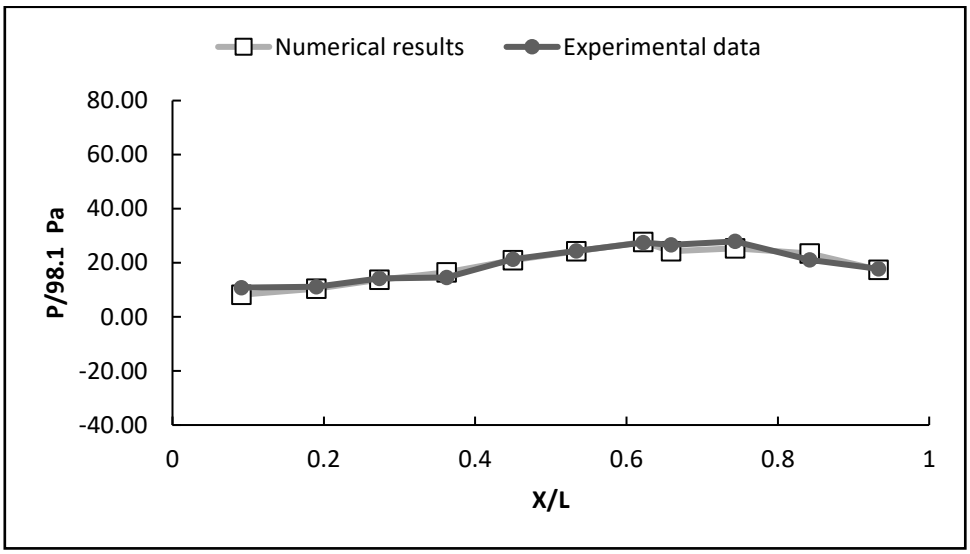
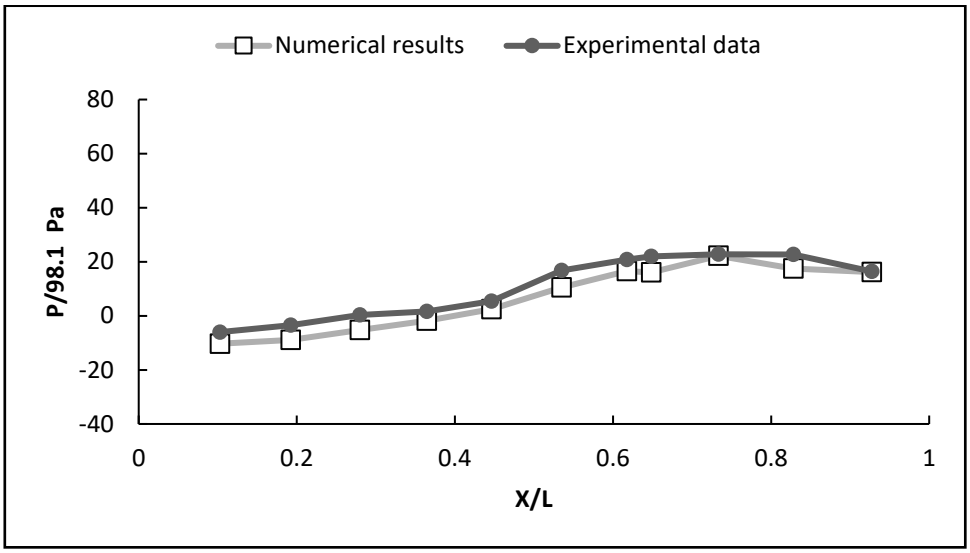


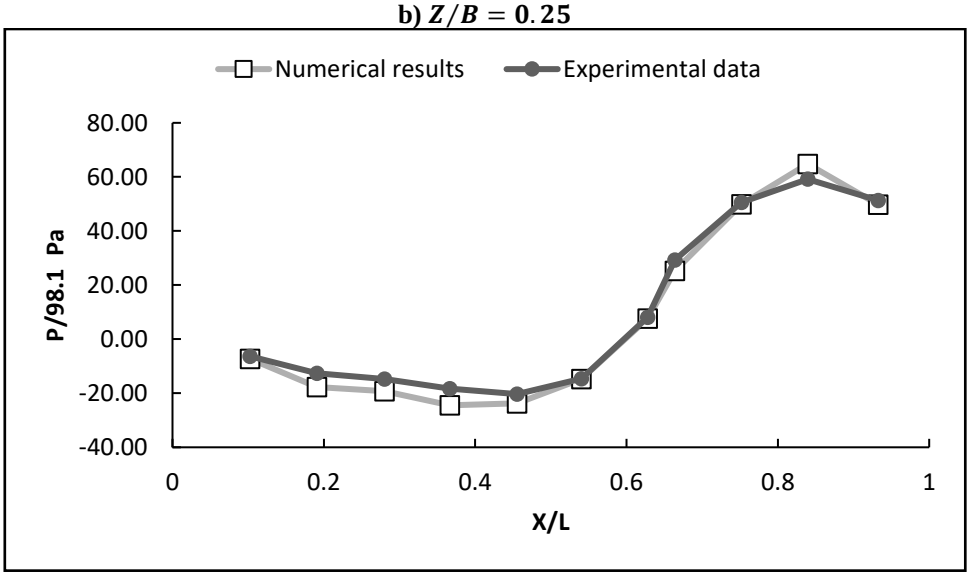

**Figure 5. The different pressure measurement points on the horizontal surface of the V-shaped stepped spillway**




**a) $Z/B = 0$**




**b) $Z/B = 0.25$**




**c) $Z/B = 0.5$**

**Figure 6. Comparison the pressure results of the present model with experimental data of Bai et al. [12] for 43rd step of a V-shaped stepped spillway with θ = 30 °**

## 4. 2. Energy dissipation rate

Various relations for energy dissipation rate on stepped spillway have been presented [22-25]. Bai et al. [25] proposed equation 8 for computing energy dissipation rate.

$$\eta = \frac{\Delta E}{E_1} \times 100\% = \frac{E_1 - E_2}{E_1} \times 100\% \quad (8)$$

where $E_1$ and $E_2$ are the total energy in the inception and ending of the stepped section, respectively; $E_1 = \Delta h + \upsilon_1^2/(2g)$, $E_2 = \upsilon_2^2/(2g)$; which $\Delta h$ is the depth difference between the two sections, and $v_1$and $v_2$ are the average velocities in the two sections. The average velocity

and height values can be obtained from the data extracted from fluent software to Tecplot software.

According to equation 8, energy dissipation rate was presented in Figure 7 for 20 cases of stepped spillways with various horizontal face angles, two roughness values (n=0.0085 and n=0.012) and with or without end sills. It can be observed that for the stepped spillway with end sills, θ=-30° and roughness value of 0.012, the highest energy dissipation of 83.8% has been obtained. As a result, energy dissipation rate increases about 0.9 to 1.94 percent with increasing roughness, and by creating an end sill on the steps, it increases about 2.8 to 3.99 percent. Moreover, the simultaneous effect of end sill and enhancement of roughness has been investigated. Results showed that the energy dissipation rate was increased about 3.26 to 5.74 percent.

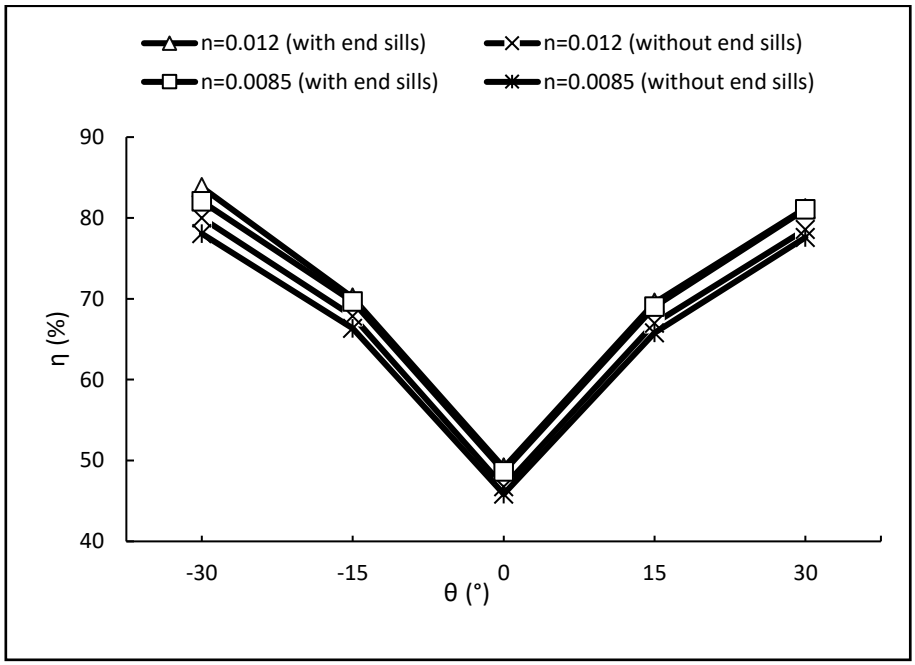



**Figure 7. Energy dissipation rate for 20 cases of stepped spillways with various horizontal face angles, two roughness and end sills**

## 4. 3. Areas of steps under negative pressure

The pressure behavior is investigated at the upside of the vertical surfaces so that negative pressures (in this study, pressure under atmospheric pressure is remarked negative) happen. Determining the amount of negative pressure and its location is very important in preventing cavitation. The negative pressure is mostly happened near both walls in a V-shaped stepped spillway and near every step's axial plane in an inverted V-shaped stepped spillway. In a traditional stepped spillway, the pressure does not alter considerably along the cross-section [18]. At first, the numerical pressure results on the horizontal surface were compared with the results of Bai et al. [12] as it is observed in Figure 6. There was a good agreement between the results. Areas of steps under negative pressure were shown in Figures 8 and 9 with the different roughness. It can be observed from the figures, the major area of a vertical step face under negative pressure. In addition, by locating end sills on the steps the negative pressure area becomes smaller. According to Figures 8 and 9, as the roughness increases, the negative pressure area decreases slightly.

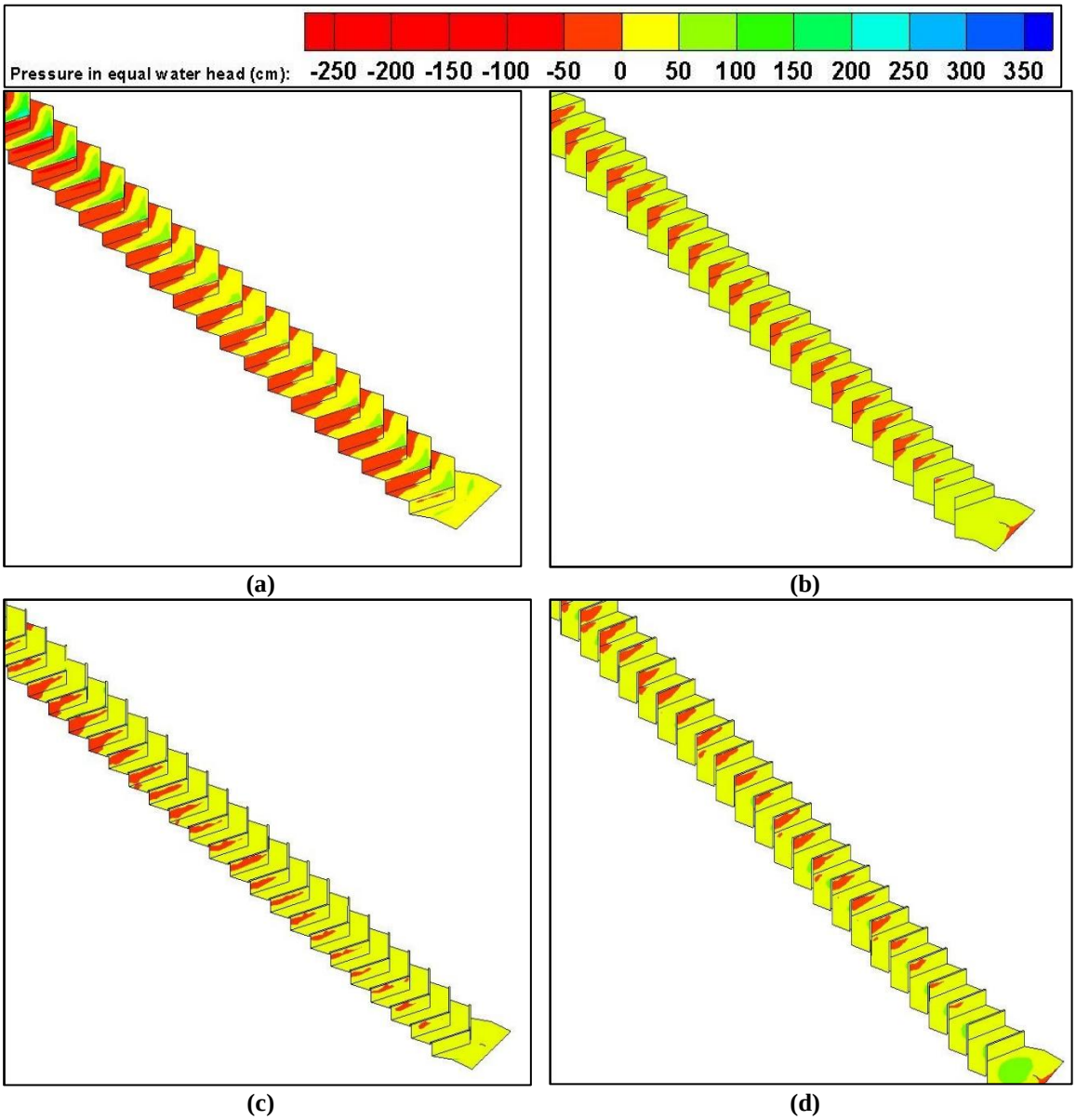



**Figure 8. Areas of steps under negative pressure in the stepped spillway with θ=30° and n=0.0085 a) without end sills b) inverted V-shaped without end sills c) V-shaped with end sills d) inverted V-shaped with end sills**

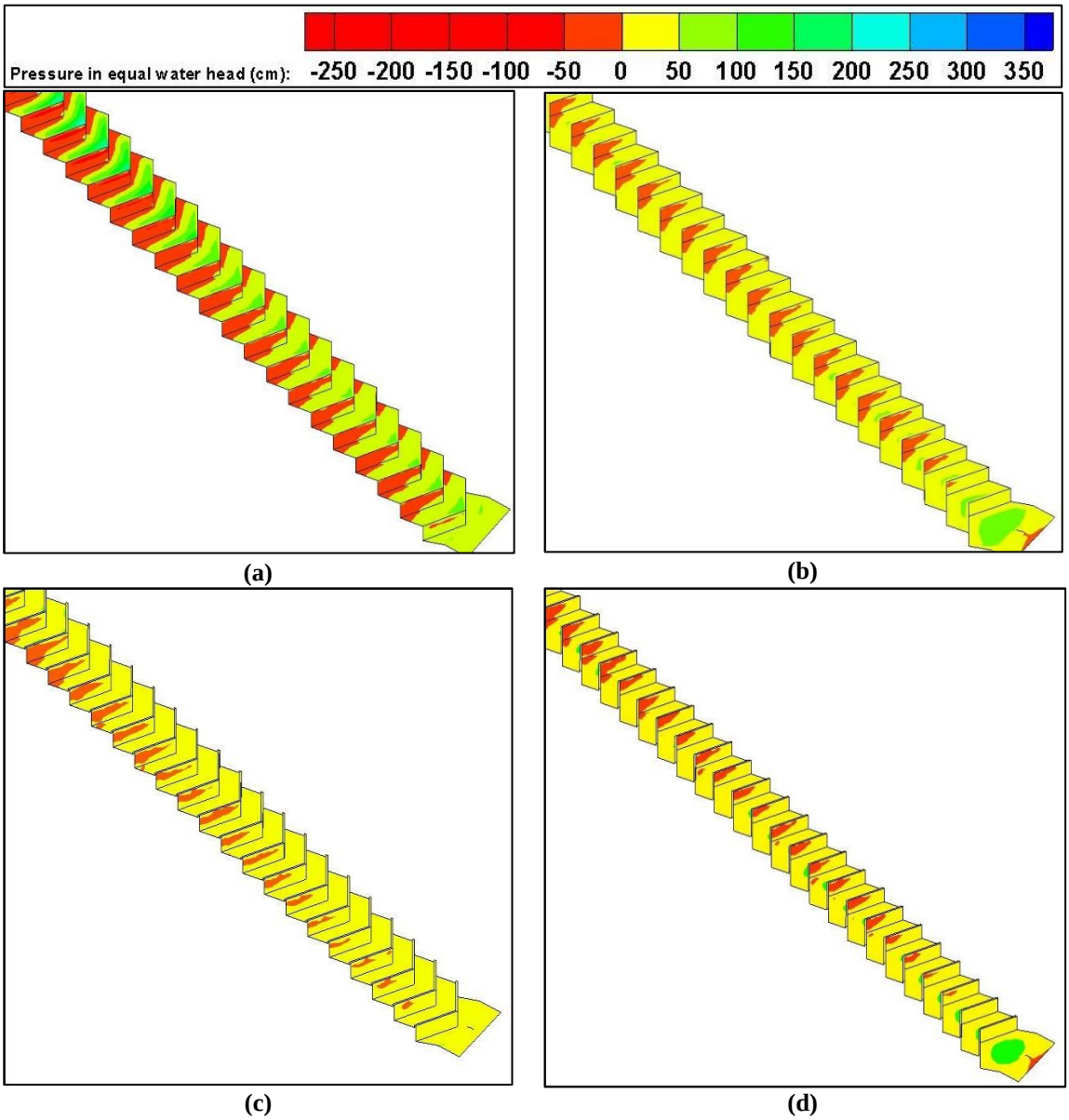



**Figure 9. Areas of steps under negative pressure in the stepped spillway with θ=30° and n=0.012 a) without end sills b) inverted V-shaped without end sills c) V-shaped with end sills d) inverted V-shaped with end sills**

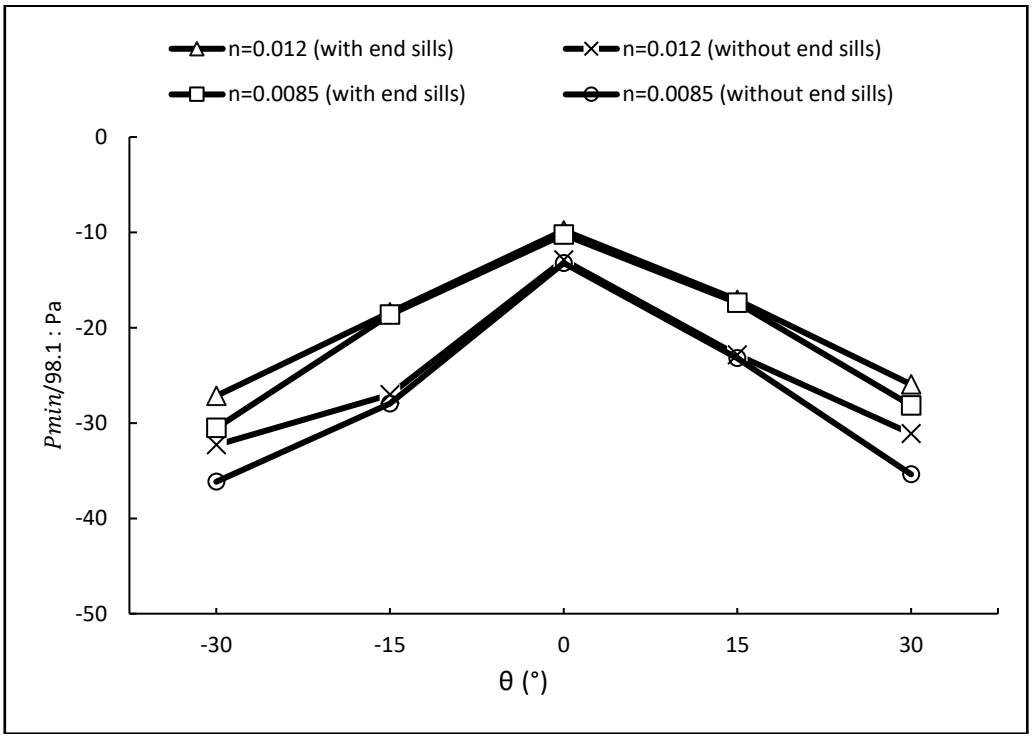



**Figure 10. Variations in the minimum negative pressure on the step surface with different horizontal face angles, two roughness values and with and without end sills.**

Figure 10 shows variations of the minimum pressures on step surface with various horizontal face angles. According to the results, the effect of end sills can be summarized as follows:

- The effect of end sills increases the minimum negative pressure in all models with each horizontal face angle.
- In θ=15° and θ=-15°, the effect of end sills is greater than other angles. For instance, in a constant value of roughness, the minimum negative pressure difference in the stepped spillway without and with end sills is 33% for θ=-15° and is 15%. θ=-30°.
- As the absolute value of horizontal face angle decreases, the effect of end sills is greater on the increasing of minimum negative pressure.

In addition, the effect of roughness can be summarized as follows:

- The roughness has negligible effect on variation of the minimum negative pressure.
- As the roughness increases, the minimum negative pressure increases.
- As the absolute value of horizontal face angle increases, the effect of roughness is greater on the increasing of minimum negative pressure.

## 5. Conclusion

In this study, the energy dissipation rate in different types of stepped spillways with various horizontal face angles has been determined using numerical modeling. Then by locating end sills and changing the roughness, variations in energy dissipation rate were investigated. Furthermore, the areas exposed to negative pressure in all spillways were identified. The results

obtained can be presented as follows:

- As the absolute quantities of the horizontal face angle increase, the energy dissipation rate increases but the negative pressure area decreases.
- The energy dissipation rate is higher in stepped spillways with end sills than those with more roughness. The negative pressure area decreases in both of stepped spillways but the increasing of minimum negative pressure is negligible in stepped spillways with more roughness compared to those with end sills.
- As the absolute values of the horizontal face angle decrease, the effect of end sills and roughness are greater on the increasing of negative pressure. In $\theta$=15° and $\theta$=-15°, its effect is greater than other angles.
- The simultaneous effect of end sill and enhancement of the roughness increase energy dissipation rate about 3.26 to 5.74 percent, which has caused the most energy dissipation rate. As a result, with better hydraulic properties, more energy dissipation rate and less negative pressure area the stepped spillway with end sills, $\theta$=-30° and roughness value of 0.012 is selected.

## References

1. Zahari, N.M., Hizami, A., Zawawi, M.H., Abas, A., Rashid, M.M.R. and Azman, A. (2019). Analysis of Different Flow Regime on Spillway Stepped Chute. International Conference on Dam Safety Management and Engineering, 577-584.
2. Chanson, H. and Toombes, L. (2002). Energy dissipation and air entrainment in stepped storm waterway: Experimental study. Journal of Irrigation and Drainage Engineering, 128(5), 305-315.
3. Haghiabi, A.H. (2012). Hydraulic characteristics of circular crested weir based on Dressler theory. Biosystems engineering, 112(4), 328-334.
4. Boes, R.M. and Hager, W.H. (2003). Hydraulic design of stepped spillways. Journal of Hydraulic Engineering, 129(9), 671-679.
5. Christodoulou, G. C. (1993). Energy dissipation on stepped spillways. Journal of Hydraulic Engineering, 119(5).
6. Chen, Q., Dai, G.Q. and Zhu F.Q. (2005). Influencing factors for the energy dissipation ratio of stepped spillways. Journal of Hydrodynamics, 17(1), 50–57.
7. Chinnarasri, C. and Wongwises, S. (2006). Flow patterns and energy dissipation over various stepped chutes. Journal of Irrigation and Drainage Engineering, 132(1), 70–76.
8. Gonzalez, C. A., Takahashi, M., and Chanson, H. (2008). An experimental study of effects of step roughness in skimming flows on stepped chutes. Journal of Hydraulic Research, 46(1), 3-10.
9. Morovati, K., Eghbalzadeh, A. and Soori, S. (2016). Numerical study of energy dissipation of polled stepped spillways. Civil Engineering Journal, 2(5), 208–220.
10. Abbasi, S. and Kamanbedast, A.A. (2012). Investigation of Effect of Changes in Dimension and Hydraulic of Stepped Spillways for Maximization Energy Dissipation. World Applied Sciences Journal, 18(2), 261-267.

11. Barani, G.A., Rahnama, M.B. and Sohrabipoor, N. (2005). Investigation of flow energy dissipation over different stepped spillways. American Journal of Applied Sciences, 117, 7123–7137.
12. Bai, Z., Wang, Y. and Zhang, J., (2018). Pressure distributions of stepped spillways with different horizontal face angles. In Proceedings of the Institution of Civil Engineers-Water Management, 171(6), 299-310.
13. Dhatrak, A. I. and Tatewar, S. P. (2014). Air entrainment and pressure fields over stepped spillway in skimming flow regime. Journal of Power and Energy Engineering, 2(4), 53–57.
14. S´anchez-Juny, M., Blad´e, E., and Dolz, J. (2007). Pressures on a stepped spillway. Journal of Hydraulic Research, 45(4), 505–511.
15. Amador, A., S´anchez-Juny, M. and Dolz J. (2006). Characterization of the nonaerated flowregion in a stepped spillway by PIV. Journal of Fluids Engineering, 128(6) 1266–1273.
16. Zhang, J. M., Chen, J. G. and Wang,Y. R. (2012). Experimental study on time-averaged pressures in stepped spillway. Journal of Hydraulic Research, 50(2), 236–240.
17. Fluent Inc. (2006). FLUENT 6.3 User's Guide. Lebanon, NH, USA.
18. Bai, Z.L., Peng, Y. and Zhang, J.M. (2017). Three-dimensional turbulence simulation of flow in a V-shaped stepped spillway. Journal of Hydraulic Engineering, 143(9), 601-701.
19. Ghazanfari-Hashemi, R.S., Namin, M.M., Ghaeini-Hessaroeyeh, M. and Fadaei-Kermani, E. (2020). A Numerical Study on Three-Dimensionality and Turbulence in Supercritical Bend Flow. International Journal of Civil Engineering, 18(3), 381-391.
20. Hirt, C.W. and Nichols, B.D. (1981). Volume of fluid (VOF) method for the dynamics of free boundaries. Journal of computational physics, 39(1), 201-225.
21. Fadaei-Kermani, E. and Barani, G.A. (2014). Numerical simulation of flow over spillway based on the CFD method. Scientia Iranica. Transaction A, Civil Engineering, 21(1), 91.
22. Ohtsu, I., Yasuda, Y. and Takahashi, M. (2004). Flow characteristics of skimming flows in stepped channels. Journal of hydraulic Engineering, 130(9), 860-869.
23. Salmasi, F. and Özger, M. (2014). Neuro-fuzzy approach for estimating energy dissipation in skimming flow over stepped spillways. Arabian Journal for Science and Engineering, 39(8), 6099-6108.
24. Parsaie, A., Haghiabi, A.H., Saneie, M. and Torabi, H. (2016). Prediction of energy dissipation on the stepped spillway using the multivariate adaptive regression splines. ISH Journal of Hydraulic Engineering, 22(3), 281-292.
25. Bai, Z., Wang, Y. and Zhang, J., (2018). Pressure distributions of stepped spillways with different horizontal face angles. In Proceedings of the Institution of Civil Engineers-Water Management, 171(6), 299-310.

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