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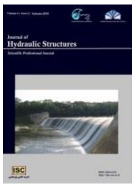

Laboratory investigation of the effect of hysteresis phenomenon of supercritical flow on the rough bed on the relative residual energy parameter

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Abstract

In the present study, the hysteretic behavior of supercritical flow that can occur in a channel near additional structures, such as changes in the bed of the channel, has been investigated experimentally and analytically. For this purpose, two diameters of the channel floor materials of 1.12 and 2 cm have been used. The critical depth ranges from 2.7 to 4.5 cm and the used flow rates range from 250 to 600 liters per minute. The flow rate increases in the primary flow and then decreases in the secondary flow into the laboratory flume. The possible flow regimes near the narrowed section are classified based on the relative depths and relative energy remaining in sections 1 and 2 as a function of the Froude number passing under the valve. The results show that by increasing the flow rate and then decreasing it, in some flow rates, two different behaviors can be seen from the flow in the same laboratory conditions. Also, the results showed that with the increase in the diameter of the bed material, the remaining relative energy values have increased a lot, so that in the primary flow, the amount of these depths indicates the subcritical regime, and in the secondary flow, with the formation of hysteresis phenomenon in some discharges, it indicates the regime Also, with the increase in the average diameter of the material, the hysteretic behavior of the flow becomes more intense.

Keywords: Hysteretic Behavior, Possible Flow Regimes, Rough Bed, Relative Depth, Relative Residual Energy.

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1. Introduction

The main goal of this research is to investigate the paradoxical behavior of supercritical flow on a rough bed. The existence of such a contradictory behavior occurs due to the phenomenon of hysteresis, which there have been very limited studies to understand and it has not been investigated for the rough bed. The occurrence of hysteresis phenomenon when the flow meets the obstacle is generally expected. So, for the same input flow, two different behaviors are observed, which is dependent on the flow cycle. Flow cycle means increasing the flow rate to a certain value and then reducing it to the initial flow rate. This phenomenon is one of the important issues that occurs in practical work in nature, but unfortunately it is not considered in the design of hydraulic structures.

The first study related to the phenomenon of hysteresis in the group that investigates the protrusion of the canal floor, goes back to the research of Abecasis and Quintella [1]. The studies in this group mostly investigate the theoretical and experimental behavior of the hysteresis phenomenon on the floor protrusion with a fixed width. The most important research in this field is related to Asteria [2], Lawrence [3], Baines and Whitehead [4] and Defina and Susin [5]. Austria [2] has described the hysteretic behavior of the flow on the channel bottom protrusion using the catastrophe theory. The theory used by him is based on the use of classic flow equations such as special energy equations along with catastrophe theory equations. Lawrence [3] investigated the permanent flow passing over the embankment of the channel floor and its different behaviors. The results showed that there may be two stable states for the same input conditions, which leads to the formation of a hysteresis loop.

Baines and Whitehead [4] investigated the hysteretic behavior of the flow on the channel bottom protrusion theoretically and experimentally. The results of theoretical and experimental studies showed that in the same conditions of the input flow, two different states are created for the stability of the flow, which indicates the existence of a hysteretic loop. Dafina and Susin [5] initially theoretically presented relations to investigate this phenomenon on the bulge and then investigated it experimentally. They defined two weak and strong reactions in the conditions of the same inlet flow for the flow, that the weak reaction is when the flow regime is supercritical and the obstacle does not affect the change of the flow regime. On the other hand, strong reaction indicates a condition where the hydraulic jump causes the flow regime to change to subcritical.

The second group of studies is related to cross-sectional contraction in the flow path, and in this regard, we can refer to the research of Akers and Bukhov [6]. They investigated the hysteretic behavior in gradual contraction theoretically and in the field. The results showed that diagonal waves of supercritical flow can be influenced by other effects such as surface tension. Defina and Viero [7] investigated the different states created by the flow in the gradual constriction. The results of their numerical and laboratory investigations showed that the friction and slope of the channel floor have an effect on the stability of the flow and can create different hysteretic loops. Sadeghfam et al. [8] investigated the hysteretic behavior of supercritical flow in the face of local narrowing of the channel by using classical hydraulic equations and catastrophe theory. Their laboratory results showed that the use of relations related to catastrophe theory along with classical relations can describe hysteretic behavior.

The third group of studies is in the field of bridge foundations and in this field we can refer to Dafina and Susin's research [9]. In this research, they used several bridge piers with different diameters and investigated the behavior of the flow in front of the used piers. Also, they presented a theoretical relationship to predict the occurrence of hydraulic hysteresis. The results showed that two different weak and strong reactions are created in the same input conditions when the flow meets the bridge foundations with narrowness. Daneshfaraz et al [10-11] also investigated the hysteretic behavior of the flow in sudden and gradual narrowings in are research.

The review of previous research shows that despite the theoretical and experimental studies conducted in connection with the hysteretic behavior of the flow, there is a need to conduct more extensive studies to investigate the unknown dimensions of the flow behavior. These unknown dimensions are more noticeable in obstacles such as bridge foundations that reduce the cross-section of the flow, which have been more or less mentioned in previous studies, but no studies have been conducted in the field of rough bed. Generally, the analytical relationships in such obstacles are less consistent with the laboratory results than in other obstacles. The reason for this could be because the flow pattern against these obstacles does not provide the possibility of creating an accurate theory in accordance with the one-dimensional flow approach. Therefore, in the present research, for the first time, analytical equations were presented to investigate the hysteretic behavior of the flow on the rough bed, and then longitudinal profiles were created in the laboratory, and the possible regimes of the flow, and then the effect of this phenomenon on the amount of relative residual energy was investigated.

2. Methodology

2.1. Description of forced hydraulic jump

Forced hydraulic jump is a process in which the supercritical flow regime created by hydraulic structures is forced to transform into the subcritical regime. In the present study, the hydraulic jump is created by two components, which are described below. The first component includes a sliding valve that is installed in the channel to measure or control the incoming flow, and it converts the subcritical flow behind the valve into the supercritical flow. The second component is the change in the bed of the channel due to the aggregates arranged in the flow path, which causes the hydraulic jump and reduces the destructive energy of the supercritical flow. Figure (1) shows different longitudinal profiles under the same input conditions. Figure (1a) shows that at one or more specific flow rates, the rough bed has an effect on the flow regime, and with the formation of a hydraulic jump on it, the supercritical flow regime turns into a subcritical flow. In other words, the flow shows a strong reaction (SI) against the rough bed. As the flow rate increases, the flow speed increases and the hydraulic jump leaves the rough bed (Figure 1b). In other words, the flow regime is on the supercritical bed and the flow shows a weak reaction (WI) against the rough bed. In this stage, the flow rate is slowly reduced until it reaches the initial flow rate. In this case, it can be seen that the behavior created on the rough bed is the same as Figure (b1) and there will be no effect of hydraulic jump on the rough bed. This state can appear in one or more specific discharges. When the flow reaches the end of the rough bed, it will leave the rough bed downstream of the channel with the critical regime.

In the above figure, E_0 is the specific energy passing through the sub-valve, E_1 is the specific energy of the flow before the rough bed, E_2 is the specific energy of the flow on the rough bed, E_{min} is the minimum specific energy, hf is the energy loss due to the rough bed, and ΔH_j is the energy loss due to the hydraulic jump. be Longitudinal drop caused by the rough bed when the condition of figure (1a) is established and the hydraulic jump occurs transfers the flow regime from supercritical to subcritical with the sign $(h_f)_L$ and when the condition of figure (1b) is established and no hydraulic jump occurred and the flow regime throughout the channel is supercritical, it is indicated by the sign $(h_f)_l$. It should be noted that when the situation in figure (1a) is established, in addition to the energy loss caused by the rough bed, there will also be an energy loss caused by the hydraulic jump.

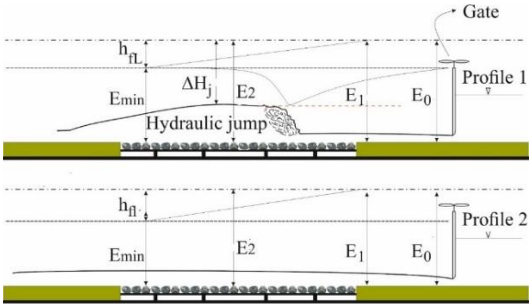

Figure 1. Longitudinal profiles of the flow created on the rough bed

2.2. Experimental equipment

The tests necessary to achieve the goals of the present research have been carried out in a laboratory flume with dimensions of 5 m in length, 0.5 m in height and 0.3 m in width, with zero longitudinal slope, walls and floor made of transparent Plexiglas. In general, 162 tests have been conducted to investigate the hysteretic behavior on the rough bed. In order to create a supercritical flow, a vertical metal valve with an opening of 2 cm, placed at a distance of 1.5 m from the inlet tank, was used. The flow entering the flume is pumped by two pumps, each with a capacity of 450 liters per minute, and the flow rate was read using rotameters installed on the pumps. In order to measure the depth of the flow, a point depth gauge with an error of ±1 mm was used. The present research has been investigated in two models with two average diameters, whose information is presented in Table 1. Figure (2) shows the schematic of the laboratory flume and the equipment installed on it, as well as the materials used in the rough bed.

Table 1. The data range and characteristics of rough bed materials

Case no.	Q (lit/min)	D_{50} (cm)	y_{cr} (cm)	y_1 (cm)	y_2 (cm)	Reynolds ($\times 10^4$)
Case 1	250~600	2.0	2.7~5.28	1.62~5.87	1.85~5.62	(5~11.2)
Case 2		1.12		1.37~4.66	1.81~4.25	

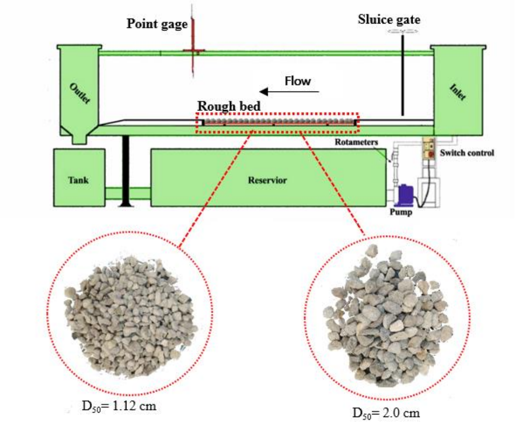

Figure 2. Schematic image of laboratory flume and rough bed materials

2.3. Longitudinal profiles

The configuration of the forced hydraulic jump in the present research includes separate parts, which are: 1- The vertical sluice gate that is installed to measure or control the flow in the channel, and its role in this research is only to form the supercritical flow, and 2- The change in the channel floor and the roughening of the bed using natural materials (sand) which causes the formation of a hydraulic jump on the rough bed and turns the supercritical flow regime into subcritical, which is very necessary to check the hydraulic parameters in this case. In Figure (1), cross-section [1] shows the starting point of the rough bed and cross-section [2] shows the flow on the rough bed and the place where the hydraulic jump is formed (in different flow rates, the position of the cross-section [2] may change). It is necessary to explain that the section [0] shows the location of the supercritical flow generating valve and the *vena cotracta* flow.

Based on the objectives of the present research and as shown in Figure (1), two types of longitudinal flow profiles can be distinguished, which are:

Profile 1: This profile shows the formation of hydraulic jump in sections [1] and [2]. Section [0] is the flow passing *vena contracta*, in which section the flow regime passing through it is supercritical. In other words, at section [1], the flow is supercritical. This flow regime is created by the upstream valve. In section [2], due to the turbulence and increase in flow depth, a hydraulic jump is formed and the flow regime is transferred to subcritical.

Profile 2: This profile indicates the absence of hydraulic jump throughout the laboratory channel. Due to the increase in flow rate, the rough bed is no longer able to maintain the hydraulic jump in its upstream, the hydraulic jump is transferred to the downstream of the channel and the regime created in sections [1] and [2] becomes supercritical. In other words, the flow regime at all points is equal to the subcritical and supercritical flow regime.

2.4. Dimensional analysis

According to Figure (1), the effective parameters for investigating the hysteretic behavior of the supercritical flow on the rough bed are as follows.

$$f_1(Q,\rho,\mu,g,a,D_{50},d,B,L,y_0,y_{cr},h_f,y_1,y_2,E_0,E_1,E_2)=0 \tag{1}$$

Where, Q is the flow rate [L^3T^{-1}], ρ is the fluid density [ML^{-3}], μ is the dynamic viscosity [$ML^{-1}T^{-2}$], g is the acceleration of gravity [LT^{-2}], a is the opening of the flow generating valve supercritical [L], D_{50} average diameter of bed material [L], d distance of valve to the beginning of rough bed [L], B width of channel [L], L length of rough bed [L], y_0 depth of flow in section 0 [L], y_{cr} critical depth [L], h_f loss due to rough bed [L], y_1 depth of flow in section 1 [L], y_2 depth of flow in section 2 [L], E_0, E_1 and E_2 according to the specific flow energy [L] in sections 0, 1 and 2. Considering the parameters ρ, y_0 and g are repeated parameters and using the Pi-Buckingham method, the dimensionless parameters are presented according to equation (2).

$$\frac{E_1}{E_0},\frac{E_2}{E_0}=f_2(Fr_0,\frac{D_{50}}{L}) \tag{2}$$

3. Results and discussion

3.1. Longitudinal profiles of flow

The longitudinal profiles of the flow on the rough bed are shown in Figures 4 and 5. As mentioned, in conducting the experiments of the present research, flow rates of 250 to 600 liters per minute (LPM) have been used in an increasing manner, and then from 600 to 250 LPM in a decreasing manner with increasing and decreasing steps of 50 LPM. Carefully in these figures, in each model with the increase and decrease of the flow rate, two different behaviors of the flow can be seen at several same flow rates, which is the hysteretic behavior of the flow in the rough bed. Table 2 and Figure 3 provide a summary of the tests performed and the hysteretic flow behavior. In the flow with increasing flow rates, the rough bed placed in the flow path causes a hydraulic jump, and during this time sections 1 and 2 are both in the subcritical regime. With the increase in flow rate, the hydraulic jump moves downstream and the flow regime becomes supercritical along the entire length of the channel. Then, with the reduction of the flow rate, the behavior that is created in the system indicates that in a certain flow rate, although not in all flow rates, two different behaviors of the flow are observed, so that the flow with a reduced flow rate created in the channel places sections 1 and 2 completely in the supercritical regime.

Table 2. Hydraulic flow parameters in model 1

Figure no.	Fr_0 *(vena contracte)*	Flow regimes (Sub/Super critical)		Profiles	Flow cycle	Case no.
		Sec. 1	Sec. 2			
3-a	2.157	Sub	Sub	P.1	Flow increasing	1
3-b	2.679	Sub	Sub		Flow increasing	
3-c	3.630	Sub	Sub		Flow increasing	
3-d	4.269	Super	Super	P.2	Increased flow caused supercritical hysteresis	
3-e	3.630	Super	Super		Decreasing flow	
3-f	2.679	Super	Super		Decreasing flow	
3-g	2.157	Sub	Sub	P.1	Decreasing flow caused subcritical hysteresis	

Figure 3 shows the regime changes and the longitudinal profiles of the flow on a rough bed with an average material diameter of 0.2 cm. By increasing the flow rate up to 350 LPM, sections 1 and 2 are in the subcritical regime. By increasing the flow rate to 400 LPM, the flow regime in the control section is transferred to supercritical. Then, with the gradual decrease of the flow rate up to 300 LPM, the control section is still in the supercritical regime, until reaching the flow rate of 250 LPM, the regime in the control section becomes subcritical.

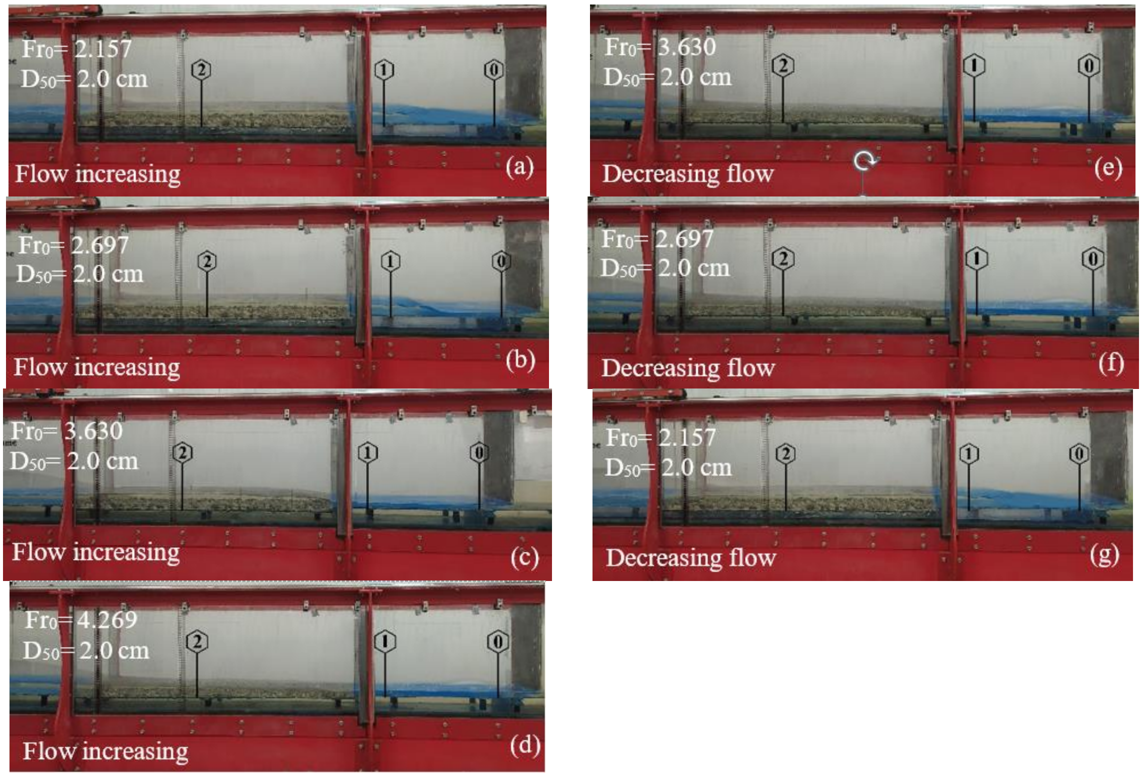

Figure 3. Case number 1 experiments to observe the phenomenon of hysteresis on a rough substrate

3.2. Relative residual energy

Figure 4 shows the changes of the relative residual energy against the Froude number of the *vena contracta* flow for model 1 with an average material diameter of 2 cm. Figure 4a, b shows the hysteretic behavior of the relative residual energy of the flow in section 1 and figures 4c, d show the said behavior in section 2.

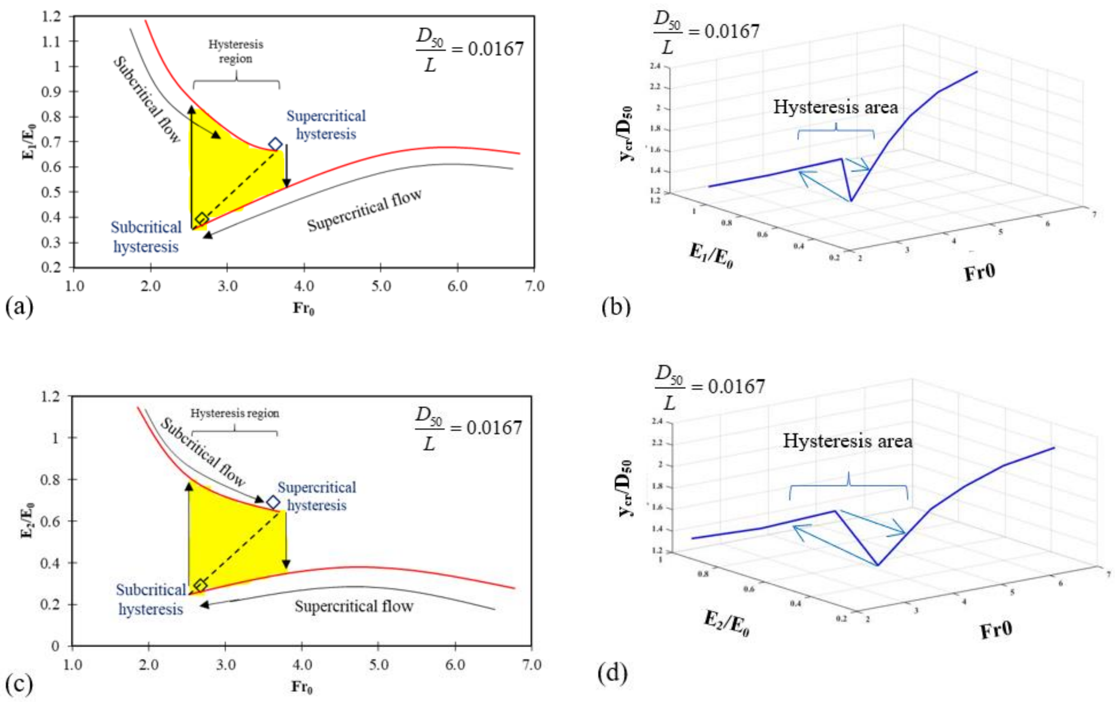

Figure 4. Reproduction of the hysteresis phenomenon by experimental data on model 1 in relative residual energy

It is carefully deduced in these figures that with the increase of the Froude number, the amount of relative remaining energy increases in both sections, but in an increasing and decreasing flow, in two flow rates with the same environmental and laboratory conditions, different behaviors are observed at flow rates of 300 and 350 LPM, it shows its hysteretic flow behavior. In these flow rates, in the initial flow, with the increase of the flow rate, a hydraulic jump is formed on the rough bed, and the flow regime becomes subcritical in the control sections. In the secondary flow, with the gradual decrease of the flow rate and when it reaches 300 and 350 liters per minute, unlike the increasing state, the hydraulic jump is not formed on the rough bed and the flow regime on the rough bed is supercritical and the Froude number in which the hysteresis phenomenon is formed and is located in the range of $2.697 \le Fr_0 \le 3.630$. Also, Figure 5 shows the changes in the hysteresis phenomenon of relative residual energy in model 2 with an average diameter of 1.12 cm. In this model, according to model 1, the phenomenon of hysteresis is formed in the Froude number range of $2.697 \le Fr_0 \le 3.630$, and its difference with model 1 is in the amount of relative energy remaining, so that in model 1, with the formation of the phenomenon of hysteresis, the amount of relative energy remaining in Sections 1 and 2 increase by 42.38% and 53.5%, respectively, and in model 2, the remaining relative energy increases by 39.5% and 45.4%. The values of the remaining relative energy increase in the present research are significant numbers and if hysteretic behavior is formed in the existing structures in nature, the downstream structures will be damaged and destroyed.

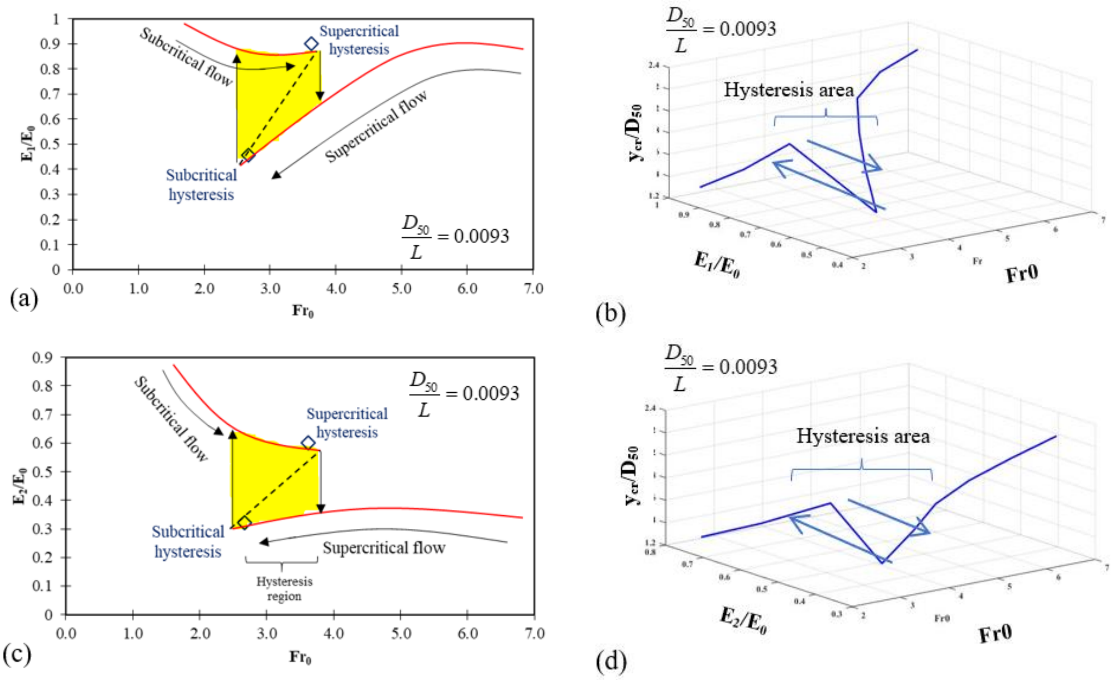

Figure 5. Reproduction of the hysteresis phenomenon by experimental data on model 2 in relative residual energy

The main reason for the formation of hysteresis phenomenon in hydraulic structures is the dependence of the flow behavior of the flow on its previous state. In this way, in the initial flow, with the increase of the flow, the flow velocity increases and with the decrease of the depth, sections 1 and 2, which were in the subcritical regime at low flow rates, become the supercritical regime. For this reason, some of the stresses created in the flow are lost with the decrease in flow rate, and some of these stresses remain as residual stress in the system, which causes the formation of hysteresis phenomenon.

The investigations carried out in the present research on the rough bed with two average diameters of the material show that with the increase of the D_{50} of the material, the residual relative energy of the flow increases. Also, the hysteretic behavior of the flow in comparing the models with each other shows that with the increase of D_{50} of the materials, the created hysteresis area has caused many changes in the amount of relative remaining energy. In other words, the hysteretic behavior of the flow on the rough bed with larger material diameter is intensified. The reason for this is that when the flow is placed on the roughness with a diameter of 2 cm, the interference of water and air in the flow of the increasing flow is high, and with the formation of the hysteresis phenomenon following the decrease of the flow, the hydraulic jump disappears and the depth of the flow decreases drastically. In other words, by increasing the average diameter of the materials, the range of hysteresis widens and causes a noticeable change in the size of the parameters. Table 3 shows the effect of hysteretic behavior on the relative residual energy parameter and the percentages of decrease or increase of these parameters. Figure 6 also shows the column chart of changes in relative residual energy values.

Table 3. Relative residual energy changes affected by hysteresis phenomenon

Parameter Model	E_1/E_0	E_2/E_0
Case 1	42.38%	53.59%
Case 2	39.50%	45.41%

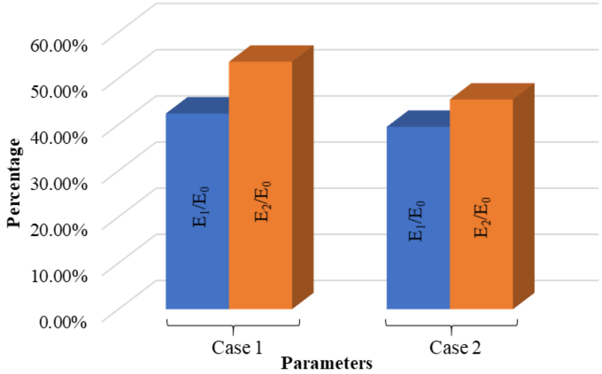

Figure 6. Column chart of relative residual energy changes affected by hysteretic behavior

4. Conclusion

The phenomenon of hysteresis in supercritical flow is one of the issues that is not well known and it is formed near the additional structures in water supply systems, water transmission lines and canals. The hysteretic behavior of the flow causes the creation of different states in the flow, and the main reason for this behavior is the dependence of the current state of the flow on its previous state, and the investigation of this phenomenon should be the attention of designers and hydraulic engineers in the hydraulic design of structures. In the present study, in order to achieve its goals, which is to investigate the conditions and reasons for the presence and absence of hydraulic jump, the formation of two different profiles and the effect of hysteretic behavior on the relative residual energy. Investigating the hysteretic behavior of the flow, the water entered the flume by creating a flow from 250 to 600 LPM and by creating a decreasing flow from 600 to 250 LPM with increasing and decreasing steps of 50 LPM.

1- In model 1, where a rough bed with D_{50} = 2.0 cm was used in the flow path, the hysteresis phenomenon was formed in the range of Froude number 2.697 and 3.63, which increased the flow rate to more than 350 LPM the supercritical regime and with the flow rate decrease to less than 300 LPM, the flow regime returns to the subcritical regime.

2- With the formation of the hysteresis phenomenon, the remaining relative energy has been affected by it, so that this phenomenon has caused many changes in the amount of the remaining relative energy as follows:

   ✓ The hysteretic behavior of the flow reduced the amount of relative residual energy in model 1 in sections 1 and 2 to 42.38 and 53.59%.

   ✓ The phenomenon of hysteresis reduced the remaining relative energy in model 2 in stages 1 and 2 by 39.5 and 45.41%.

   ✓ The increase in the average diameter of the substrate material aggravated the hysteretic behavior in the control section.

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