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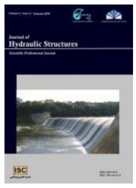

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Feasibility study of fuzzy method in slope stability analysis of earth dams with respect to the uncertainty of geotechnical parameters

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Abstract

The stability of earth dams is assessed through safety factors, indicating stability if they exceed one. Due to soil property uncertainties, fuzzy logic tools seem suitable for slope stability analysis. Uncertainties in parameters like unit weight, cohesion, and internal friction angle can be encompassed by fuzzy set theory. This study employs fuzzy set theory to analyze slope stability factor of safety, considering the varied materials and soils in earth dams. Information and parameters were gathered, and slopes were modeled using Slide (v. 6) software. Shear strength parameters and safety factors were categorized based on results and expert opinions, defining ranges for each. MATLAB software applied fuzzy logic rules to relate inputs (unit weight, cohesion, friction angle) to the output, factor of safety. Comparing results from probabilistic and fuzzy methods revealed close numerical alignment. The fuzzy method, with adaptable rules accommodating different conditions, yielded quicker and more accurate safety assessments, assuming specific data inputs. Overall, the fuzzy approach offers flexibility, facilitating quicker and more accurate determinations of safety factors, albeit requiring specific data assumptions.

Keywords: Slope Stability; Fuzzy System; Probabilistic Analysis; Fuzzy Rules

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1. Introduction

Slope analysis is critical in the process of developing an earth dam. In reality, the slope stability of earth dams is generally assessed using safety factors (FS), and dam slopes are deemed stable if these factors exceed one [1]. However, there is a significant amount of uncertainty in predicting the slope stability. These uncertainties are divided into two categories:

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those connected with material characteristics and those associated with layer geometry and geotechnical profile [2-4]. Artificial intelligence approaches have been widely employed in recent decades to simulate various geotechnical problems [5-7], owing to their capacity to account for uncertainty. Analyzing the stability of slopes with artificial intelligence methods has also been one of the topics of interest. For example, Khajehzadeh et al. [8] suggested an effective intelligent method based on artificial neural networks (ANN) to estimate the factor of safety of homogeneous slopes under static and dynamic loading. Paliwal et al. [9] also employed an artificial neural network to forecast the stability of a seam and the slope of the remaining soil in the Himalayas.

Among these methods, fuzzy logic method is a very useful and practical method for analyzing complex problems with high uncertainties. Fuzzy systems are a popular technique that can be employed in the geotechnical problems specially in the stability analysis of slopes and earth dams [10]. Fuzzy theory may be used to assess known uncertainties in slope stability evaluation as an alternative to probabilistic approaches. Fuzzy systems employ variables with some uncertainty, which are subsequently used in a constitutive model for every particular material. Fuzzy numbers theory has already been applied to slope stability evaluations in a few works [11-16]. Peng and Huang [14] fully analyzed the safety of earth dams using fuzzy categorization and generated safety assessment indices by analyzing many elements influencing operational and general safety in earth dams. Yang [17] applied fuzzy assessment to determine the safety performance of an earth dam using the analytic hierarchy process (AHP) and concluded that the earth dams might be designed with a combination of natural, social, and economic elements. Lim et al. [18] used three-dimensional analysis to assess slope stability and make recommendations for future evaluations. Fattahi [19] investigated slope stability prediction utilizing soft computing approaches such as the adaptive neuro-fuzzy inference system (ANFIS), which is based on clustering. The results shown that the ANFIS-SCM (subtractive clustering method) model is a valid system modeling approach for predicting slope stability. Haghshenas et al. [20] employed fuzzy multi-criteria decision making (FMCDM) methodologies to assess the importance of each component in dam construction projects. Due of the uncertainty in each factor, they employed a fuzzy inference approach to quantify their likelihood and severity. The outcomes of this analysis indicated that the risk of design faults is the biggest of all the examined hazards.

The purpose of this research is to evaluate the feasibility of the fuzzy method in analyzing the stability of the slope of earth dams with regard to the uncertainty of geotechnical parameters. The reason why the fuzzy method is used in this study is that there are imprecise facts about the limits of the parameters. However, the probabilistic method which is based on a set of random states of parameters and indicates the chance of a particular state cannot be used alone, but in combination with the fuzzy method, they can complement each other and use both methods. Therefore, it is necessary to use a probabilistic method to analyze slope stability so that these two methods complement each other. Since the shear resistance parameters of different materials used in earth dams are very different, a range of these parameters is used to obtain the reliability factor in reality. So, in this research, stable slopes for earth structures like earth dams and embankments are modeled using the fuzzy method. If a suitable method can be found to determine whether the slope is stable or not in this method, it is possible to determine the stability of slopes without the need of modeling and using this theory.

2. Analysis Method

To achieve the objectives of this study, the following actions have been taken:

- ✓ Data collection and gathering of material parameters necessary for analyzing the stability of embankment slopes of an earth dam through literature review and calculation of statistical parameters of the data.
- ✓ Modeling and stability analysis of slope structures using a probabilistic approach in Slide (v. 6) software with varying slopes within the range of material parameters.
- ✓ Investigation of variations in the factor of safety with different parameters.
- ✓ Classification of shear strength parameters and obtained factors of safety from the previous stage analysis using expert opinions, and determination of ranges for each of them.
- ✓ Calculation of statistical parameters for different classifications.
- ✓ Utilization of fuzzy logic method in MATLAB software and definition of rules to relate analysis parameters (input data) to the factor of safety (output data).
- ✓ Comparison of results obtained from the probabilistic model with those obtained from the fuzzy system and determination of the best fuzzy rules and conclusions.

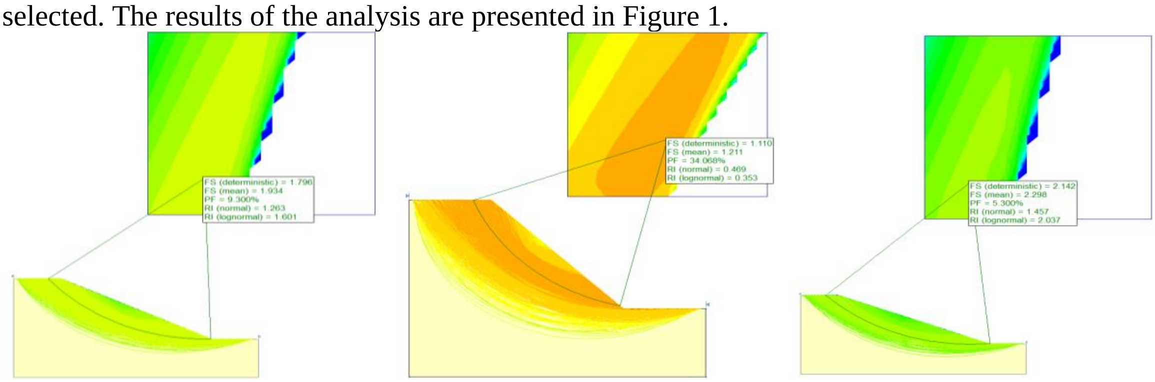
In this study, initially, all the necessary information and parameters, i.e., the cohesion (C), unit weight (γ), and internal friction angle (ϕ), for analyzing the stability of embankment slopes of an earth dam have been collected from various sources, as observed in Table 1.

Table 1. Values of shear strength parameters of materials from literature review

Ingredients	γ (kN/m^3)	C (kN/m^2)	ϕ (deg.)	Reference
Tar Sand	91-94(PCF)	15-37	18-24	[21]
Concrete and Asphalt	14.7-17.8	38-80	39.1-45	[22]
Cohesive Soil and Crushed Concrete	16.77-17.75			[23]
Weak soil and Concrete Aggregate	12.16-14.42			[24]
Concrete and Soil	16.77			[25]
Polypropylene Strips and Lateritic Soils	16.6-17.8	53-54	21.0-32.0	[26]
Substrate (sand, silt and clay) and glass	17.36			[27]
Glass and Subgrade Soil	13.97-16.18			[28]
Glass powder and Subgrade Soil	18-20	42.7-106.4	27.4-43.5	[29]
Fly Ash	15.4-18.14			[30]
Chalk		20-131	30-42.0	[31]
Dumped Rock			35-50	[32]
Rock			20-65.0	[33]
Rockfill			35-52.0	[34]
Rock	25.5	10-50.0		[35]
Rock	20.6			[35]
Rock	23	11.1-117.66	35.69-45.94	[36]
Rock	8.8-23.2			[37]
Gravel Mixtures			32-45.0	[38]
Sand and Gravel Mixtures	15.89-16.28		34-45.0	[38]
Sand and Gravel Mixtures	23	15-59.4	31-67	[39]
Sand and Gravel Mixtures			23.7	[40]
Sand and Gravel Mixtures			32-53	[41]
sand mixtures	15.89		32	[38]
sand mixtures			26.7-27	[40]
sand mixtures	14.47-17.12		18.3-41.6	[42]
sand mixtures	13.6	9.8	13.6-21	[43]
Clay mixtures	11.87	13-98	0-34	[44]
Clay-Gravel Mixture			35-50.0	[45]
Silty Sand			29-37	[46]

The Slide software (v. 6) was utilized for probabilistic stability analysis of slopes. This software enables extensive probabilistic analyses while allowing users to select the type of probabilistic analysis they prefer. Additionally, users have the capability to assign statistical distribution types for each of the input parameters, including material properties, groundwater levels, and loads. Sensitivity analysis in this software allows users to determine the effect of each variable on the factor of safety.

After collecting the desired parameters from each of the available materials, their statistical parameters, including Maximum, Minimum, Average, Median, Variance, and Standard Deviation, were calculated. Then, the slope was modeled in the Slide software. Since the slope is completely heterogeneous, probabilistic slope analysis was performed in the Slide software. For this purpose, the GLE/Morgenstern method was used for probabilistic analysis as it considers both the force equilibrium and moment equilibrium in analysis and design, which is the best method. However, other methods such as Bishop and Janbu do not consider both of these equilibrium states in their analysis. The slip surface was considered circular, the search method was set to Grid Search, the design option was selected as Statistic, Monte Carlo sampling method was used, and 1000 samples were chosen by default. The analysis was performed on three different slopes: 1 to 1, 1 to 2, and 1 to 2.5, with a constant slope height of 80 meters. Then, the relative minimum and maximum values, mean, and standard deviation for the cohesion (C), unit weight (γ), and internal friction angle (ϕ) parameters, which had been previously calculated, were defined as parameters with uncertainty, and the normal distribution curve was selected. The results of the analysis are presented in Figure 1.



Slope 1 to 1 **Slope 1 to 2** **Slope 1 to 2.5**

Figure 1. Results of modeling and probabilistic analysis of different slopes using Slide

In Table 2, the results of the analysis are also provided. According to the results obtained from the analysis of the three slopes in Table 2, slopes 1 to 2 and 1 to 2.5 are completely stable and have high factors of safety. Therefore, slope 1 to 1, which has lower factors of safety and is closer to 1, is considered as the critical slope for design purposes.

Table 2. summarizes the results of modeling with different slopes.

Slope	FS (deterministic)	FS (mean)	PF %	RI (normal)	RI (lognormal)
1:1	1.11	1.211	34.068	0.469	0.353
1:2	1.796	1.934	9.3	1.263	1.601
1:2.5	2.142	2.298	5.3	1.457	2.037

FS (deterministic): The deterministic factor of safety of the slope obtained from conventional analysis with inputting average parameters.
FS (mean): The probabilistic stability factor of the slope obtained from probabilistic analysis.
PF: The percentage of failed samples to the total number of samples.
RI: Reliability index (for normal distribution, represents the number of standard deviations that separate the mean Factor of Safety, from the critical Factor of Safety (= 1).

Following that, a scatter plot for each of the unknown parameters relative to the factor of safety was plotted, and the data for each output were extracted from Excel to compare the factors of safety for all samples for each of the unit weight, cohesion, and friction angle parameters. Since the factors of safety for parameters were the same in each sample, it was concluded that the parameter values simultaneously and in conjunction with other parameters provided a consistent factor of safety. For example, for the first sample, the factor of safety for all cohesion, friction angle, and unit weight parameters was 0.146, and similarly, the factors of safety for these three parameters were the same for other samples. Therefore, 1000 samples obtained from the probabilistic scenarios provided by the software were considered as real-life samples.

Taking into account real conditions, experiences, and consulting experts, the factor of safety was divided into three categories. The considered categories for the factor of safety were as follows: The first category included factors of safety less than 1, indicating instability (F). The second category included factors of safety greater than 1 and less than 1.5, suggesting potential stability (PS). The third category included factors of safety greater than 1.5, indicating definite stability (S). Based on these three categories, the unit weight, cohesion, and friction angle parameters were also divided into three categories within these ranges, considering low (L), medium (M), and high (H) states for each.

The maximum, minimum, mean, and median values for each parameter category were calculated and are presented in Table 3. Additionally, scatter plots were created for each parameter within each category to facilitate analysis. The scatter plot depicting cohesion versus factor of safety is presented in Figure 2, the plot for friction angle versus factor of safety is displayed in Figure 3, and the plot illustrating unit weight versus factor of safety is shown in Figure 4.

Table 3. Statistical values of the classified shear strength parameters of samples

Parameter		**1< FS**	**FS < 1.5≤1**	**FS ≥ 1.5**
C (kN/m^2)	Median	32.65	54.58	67.93
	Average	34.88	55.32	69.59
	Min	0.06	0.41	1.07
	Max	97.24	170.07	143.53
ϕ (deg.)	Median	20.99	33.2	44.85
	Average	20.62	32.67	43.81
	Min	0.72	11.49	19.46
	Max	36.56	49.01	57.34
γ (kN/m^3)	Median	16.9	16.97	15.93
	Average	16.89	17.04	16.11
	Min	11.95	12.12	11.91
	Max	24.46	24.25	23.69
FS	Median	0.76	1.24	1.73

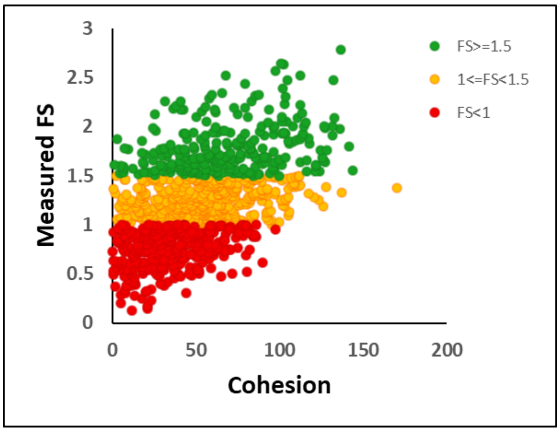


Figure 2. The values of the obtained safety factors based on samples with different cohesions

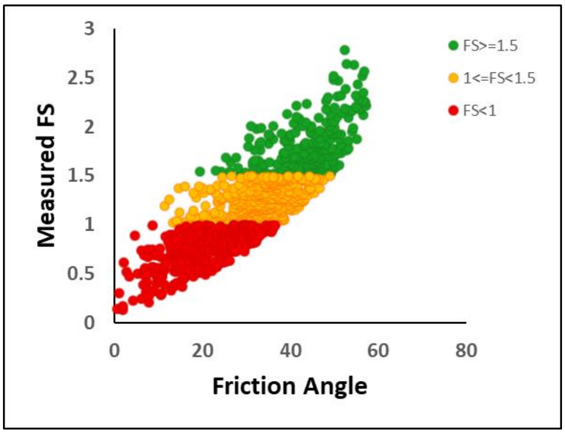


Figure 3. The values of the obtained safety factors based on samples with different friction angles

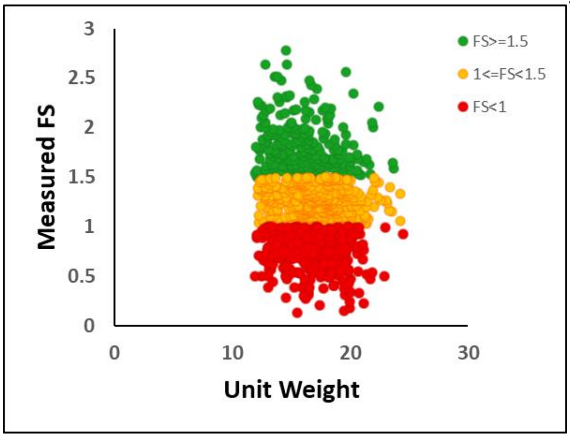


Figure 4. The values of the obtained safety factors based on samples with different unit weights

As observed in Figure 4, the variations in unit weight were nearly constant across all three categories. Therefore, only one average condition for unit weight was considered. However, in the cohesion and friction angle plots, as expected, it was observed that with an increase in these parameters, the safety factor also increased. Nevertheless, many values in these categories overlapped with each other. This implies that, for instance, a range of values from the first category, yielding safety factors less than 1, might also fall within the second or third category, where their safety factors are greater than 1 or 1.5, respectively. In such situations, analyzing the safety factor becomes challenging because it's not clear which specific cohesion, unit weight, or friction angle value may result in what safety factor. In these circumstances, fuzzy logic theory is highly applicable and provides an appropriate solution. These overlaps and uncertain boundaries essentially embody the concept of fuzzy logic, which has been introduced for handling uncertain boundaries and ambiguous parameters.

2.1. Fuzzy Stability Analysis

To utilize fuzzy theory, MATLAB software is necessary. The fuzzy system comprises inputs, rules, and outputs. Here, the inputs include unit weight, cohesion, and friction angle, while the output is the factor of safety. The considered ranges are explained further in Table 4.

Table 4. Information of the defined fuzzy system for models

Input	C (kN/m^2)	L	Type	trapmf
		M	Type	trimf
		H	Type	trapmf
	ϕ (deg.)	L	Type	trapmf
		M	Type	trimf
		H	Type	trapmf
	γ (kN/m^3)	M	Type	trimf
Output	FS	F	Type	trapmf
		PS	Type	trimf
		S	Type	trapmf
Methods	And method			min
	Or method			max
	Implication			min
	Aggregation			max
	Defuzzification			centroid

trapmf = Trapezoidal membership function
trimf = Triangular membership function
L=low, M = medium and H =high
F = instability
PS = potential stability
S = definite stability

The ranges for each category were selected based on the scatter plots from Figures 2 to 4. To do this, scattered points were disregarded, and boundary points for each category were considered to define its range. In these circumstances, some points also overlapped with other categories. The membership degree of each point to each category, indicating the extent to which it belongs to that category, was determined using fuzzy membership functions. A fuzzy inference system (FIS) consists of 6 stages, which are:

1. Input Data: In this stage, the input parameters such as unit weight, cohesion, and friction angle, which have been predetermined, are specified as input parameters.
2. Fuzzification: In this stage, a specific function is determined for each parameter, and the ranges considered for the functions are also indicated on the graph. In this part, the membership degree of the input functions is determined. Among the functions that are most commonly used are triangular, trapezoidal, Gaussian, sigmoidal, π-shaped, and S-shaped.
3. Implication: In this stage, using a set of rules, we relate the input data to the output data. To define these rules, past experiences, expert opinions, and comparison with actual results are utilized. If necessary, adjustments to the rules can be made to reduce errors, ensuring that fuzzy analysis results closely match real-world outcomes. Conditions can be defined using "and" and "or" operators between sentences. The "and" operator (fuzzy intersection), also known as "T-norm," has several methods, including "min" and "prod," which consider the minimum and product of states, respectively. Similarly, the "or" operator (fuzzy union), also known as "S-norm," has two methods: "max" and "probor," which consider the maximum value and the probabilistic sum of functions, respectively. The Implication method itself can also have two modes: one using the minimum value and the other using the product of functions.
4. Aggregation: In this stage, the outputs of all rules are combined with each other to form a composite fuzzy set. Aggregation of output functions is also performed using various

methods, including "max," "sum," and "probor," which respectively consider the maximum value, sum, and probabilistic sum of functions.

5. Defuzzification: Using specific methods, the fuzzy output function obtained from the previous stage, which is in the form of a fuzzy graph and lacks a clear concept, can be transformed into a definite non-fuzzy number. Some of the most important methods for defuzzifying the output function includes centroid method, median, mean of maximum, maximum value, and minimum of maximum value. However, besides these, other functions can also be defined and used.
6. Output: The numerical value obtained after defuzzification provides a degree of membership from the output functions based on the defined conditions and selected methods, which essentially represents the required confidence level.

In this study, various scenarios for each of these methods and conditions were considered. Many scenarios were discarded due to significant discrepancies with the original results obtained from slope stability analysis in the Slide software, and only six scenarios with results close to each other were investigated and compared.

Conditions Considered in the Fuzzy System

The functions chosen for this study were triangular and trapezoidal functions. The reason for selecting these two functions is their wide applicability, ease of use, and the possibility of selecting the ranges of functions. Although using curved functions such as Gaussian and bell-shaped curves might potentially provide higher accuracy if the curves are considered symmetrically, any change in one side of the curve would affect the other side, making the selected functions asymmetric. Therefore, it would be challenging to determine the desired range accurately. In Figure 5, the components of the fuzzy system and parameters are specified.

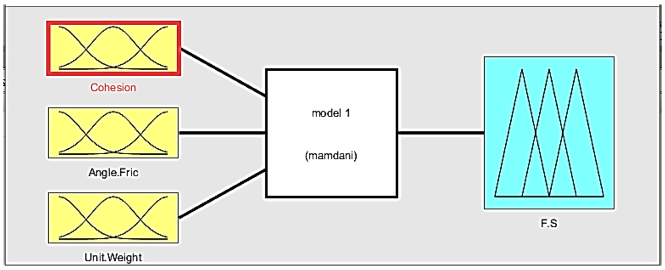
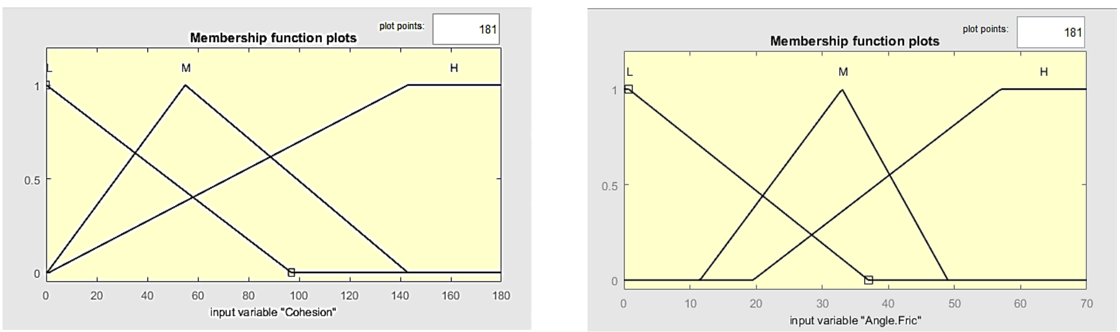
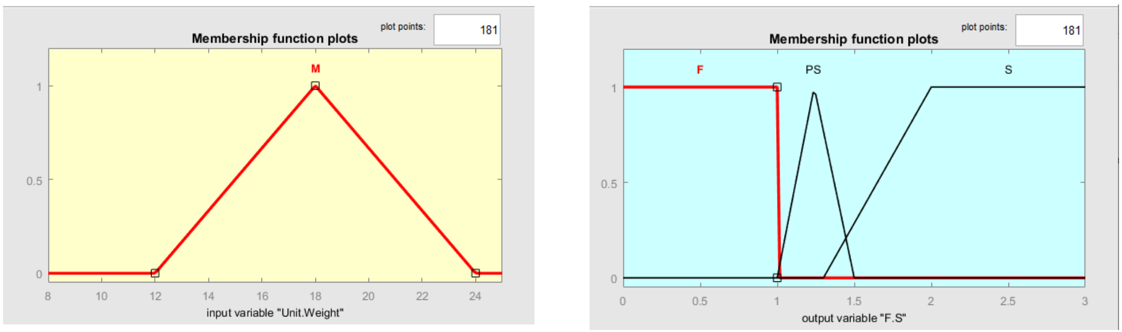


Figure 5. Parameters defined for the fuzzy system



(a) **(b)**



(c) (d)

Figure 6. Functions and Parameter Ranges for (a) Cohesion, (b) Internal Friction Angle, (c) Unit Weight and (d) Factor of Safety

In Figure 6, the selected functions and the considered ranges for each function have been specified. The membership degree of each parameter can be determined in these functions. In this way, the y-axis represents the membership degree function from 0 to 1, while the x-axis denotes the range of the data. As previously mentioned, the selection of the ranges for each category is based on the obtained graphs from Figures 2 to 4, adjusted from the initial data. In triangular functions, the apex of the triangle represents the mean parameter value in that category. Additionally, the points to the left and right of each category represent the minimum and maximum parameter values in that category, respectively. As observed, the membership degree of values before the mean increases until it reaches a membership degree of 1, then decreases after the mean until it reaches 0. Points between two categories have a membership degree in both categories, decreasing in one category and increasing in the other. It is also possible for the membership degree in both categories to be 0.5. In the categories with the highest values, a triangular function with infinite properties is typically used. This means that if the parameter value exceeds a certain limit, its membership in that category will be complete, with a membership degree of 1. This condition implies that the infinite triangular function can also be used for the low (L) state, meaning that even for values below a certain threshold, its membership in this category will be 1. For the conditions and rules of the fuzzy system, attempts and errors were made in ranges close to reality, and ultimately, the combination of these conditions, which had less error compared to others, was selected.

Table 5. The range of each function defined for the models

I/O	Parameter	Function type	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Input	C (kN/m^2) Fig. 6(a)	L = trapmf	[-inf -inf 0.06 97]	[-inf -inf 0.06 97]	[-inf -inf 0.06 97]	[-inf -inf 0.06 97]	[-inf -inf 0.06 97]	[-inf -inf 0.06 97]
		M = trimf	[0.4 55 143]	[0.41 55.32 170]	[0.41 55.32 170]	[0.41 55.32 170]	[0.41 55.32 170]	[0.41 55.32 170]
		H = trapmf	[1 143 inf inf]	[1.07 143 inf inf]	[1.07 143 inf inf]	[1.07 143 inf inf]	[1.07 143 inf inf]	[1.07 143 inf inf]
	ϕ (deg.) Fig. 6(b)	L = trapmf	[-inf -inf 0.72 37]	[-inf -inf 0.72 36.56]	[-inf -inf 0.72 36.56]	[-inf -inf 0.72 36.56]	[-inf -inf 0.72 36.56]	[-inf -inf 0.72 36.56]
		M= trimf	[11.5 33 49]	[11.5 32.67 49]	[11.5 32.67 49]	[11.5 32.67 49]	[11.5 32.67 49]	[11.5 32.67 49]
		H = trapmf	[19.5 57 inf inf]	[19.46 57.34 inf inf]	[19.46 57.34 inf inf]	[19.46 57.34 inf inf]	[19.46 57.34 inf inf]	[19.46 57.34 inf inf]
	γ (kN/m^3) Fig. 6(c)	M = trimf	[12 18 24]	[11.91 16.68 24.46]	[11.91 12.12 23.69 24.46]	[11.91 12.12 23.69 24.46]	[11.91 12.12 23.69 24.46]	[11.91 12.12 23.69 24.46]
Output	FS Fig. 6(d)	F = trapmf	[-inf -inf 1 1]	[-inf -inf 1 1.1]	[-inf -inf 1 1.1]	[-inf -inf 1 1.1]	[-inf -inf 1 1.1]	[-inf -inf 1 1.1]
		PS = trimf	[1 1.24 1.5]	[1 1.24 1.5]	[1 1.24 1.5]	[1 1.24 1.5]	[1 1.24 1.5]	[1 1.24 1.5]
		S = trapmf	[1.3 2 inf inf]	[1.3 2 inf inf]	[1.3 2 inf inf]	[1.3 2 inf inf]	[1.3 2 inf inf]	[1.3 2 inf inf]

After several trials and errors, six models were chosen, which had similar results and were better than other analyses. Tables 5 and 6 provide information on the methods, rules applied, and changes made in the six fuzzy analyses performed.

Table 6. Defined rules for models

Model	Rule No.	Rules	Model	Rules
	1	If (C is L) and (ϕ is L) and (γ is M) then (FS is F)		If (C is L) and (ϕ is L) and (γ is M) then (FS is F)
	2	If (C is L) and (ϕ is M) and (γ is M) then (FS is F)		If (C is L) and (ϕ is M) and (γ is M) then (FS is F)
	3	If (C is L) and (ϕ is H) and (γ is M) then (FS is PS)		If (C is L) and (ϕ is H) and (γ is M) then (FS is PS)
	4	If (C is M) and (ϕ is L) and (γ is M) then (FS is F)		If (C is M) and (ϕ is L) and (γ is M) then (FS is PS)
1	5	If (C is M) and (ϕ is M) and (γ is M) then (FS is F)	4	If (C is M) and (ϕ is M) and (γ is M) then (FS is PS)
	6	If (C is M) and (ϕ is H) and (γ is M) then (FS is PS)		If (C is M) and (ϕ is H) and (γ is M) then (FS is PS)
	7	If (C is H) and (ϕ is L) and (γ is M) then (FS is PS)		If (C is H) and (ϕ is L) and (γ is M) then (FS is PS)
	8	If (C is H) and (ϕ is M) and (γ is M) then (FS is PS)		If (C is H) and (ϕ is M) and (γ is M) then (FS is PS)
	9	If (C is H) and (ϕ is H and (γ is M) then (FS is S)		If (C is H) and (ϕ is H and (γ is M) then (FS is S)
	1	If (C is L) and (ϕ is L) and (γ is M) then (FS is F)		If (C is L) and (ϕ is L) and (γ is M) then (FS is F)
	2	If (C is L) and (ϕ is M) and (γ is M) then (FS is F)		If (C is L) and (ϕ is M) and (γ is M) then (FS is F)
	3	If (C is L) and (ϕ is H) and (γ is M) then (FS is PS)		If (C is L) and (ϕ is H) and (γ is M) then (FS is PS)
	4	If (C is M) and (ϕ is L) and (γ is M) then (FS is F)		If (C is M) and (ϕ is L) and (γ is M) then (FS is PS)
2	5	If (C is M) and (ϕ is M) and (γ is M) then (FS is PS)	5	If (C is M) and (ϕ is M) and (γ is M) then (FS is PS)
	6	If (C is M) and (ϕ is H) and (γ is M) then (FS is S)		If (C is M) and (ϕ is H) and (γ is M) then (FS is PS)
	7	If (C is H) and (ϕ is L) and (γ is M) then (FS is PS)		If (C is H) and (ϕ is L) and (γ is M) then (FS is PS)
	8	If (C is H) and (ϕ is M) and (γ is M) then (FS is S)		If (C is H) and (ϕ is M) and (γ is M) then (FS is PS)
	9	If (C is H) and (ϕ is H and (γ is M) then (FS is S)		If (C is H) and (ϕ is H and (γ is M) then (FS is S)
	1	If (C is L) and (ϕ is L) and (γ is M) then (FS is F)		If (C is L) and (ϕ is L) and (γ is M) then (FS is F)
	2	If (C is L) and (ϕ is M) and (γ is M) then (FS is F)		If (C is L) and (ϕ is M) and (γ is M) then (FS is F)
	3	If (C is L) and (ϕ is H) and (γ is M) then (FS is PS)		If (C is L) and (ϕ is H) and (γ is M) then (FS is PS)
	4	If (C is M) and (ϕ is L) and (γ is M) then (FS is PS)		If (C is M) and (ϕ is L) and (γ is M) then (FS is PS)
3	5	If (C is M) and (ϕ is M) and (γ is M) then (FS is PS)	6	If (C is M) and (ϕ is M) and (γ is M) then (FS is PS)
	6	If (C is M) and (ϕ is H) and (γ is M) then (FS is PS)		If (C is M) and (ϕ is H) and (γ is M) then (FS is PS)
	7	If (C is H) and (ϕ is L) and (γ is M) then (FS is PS)		If (C is H) and (ϕ is L) and (γ is M) then (FS is PS)
	8	If (C is H) and (ϕ is M) and (γ is M) then (FS is PS)		If (C is H) and (ϕ is M) and (γ is M) then (FS is PS)
	9	If (C is H) and (ϕ is H and (γ is M) then (FS is S)		If (C is H) and (ϕ is H and (γ is M) then (FS is S)

3. Analysis of Results

Figure 7 compares the factor of safety obtained from the fuzzy analyses with the estimated and initial factor of safety. Statistical parameters were utilized for a more accurate comparison. These parameters include:

- RMSE (Root Mean Square Error): The square root of the mean of the squared differences between estimated and simulated values, providing an indication of the typical deviation.
- CD (Coefficient of Determination): Indicates the proportion of the variance in the simulated values that is predictable from the estimated values.
- EF (Modeling Efficiency): Measures the efficiency of the modeling process.
- CRM (Coefficient of Residual Mass): Measures the remaining mass coefficient.
- SE (Standard Error): Measures the typical deviation of the estimated values from the simulated values.
- RE (Relative Error): Indicates the relative deviation between estimated and simulated values.
- COREE (Correlation): Also known as the correlation coefficient (r), measures the strength and direction of the linear relationship between estimated and simulated values.

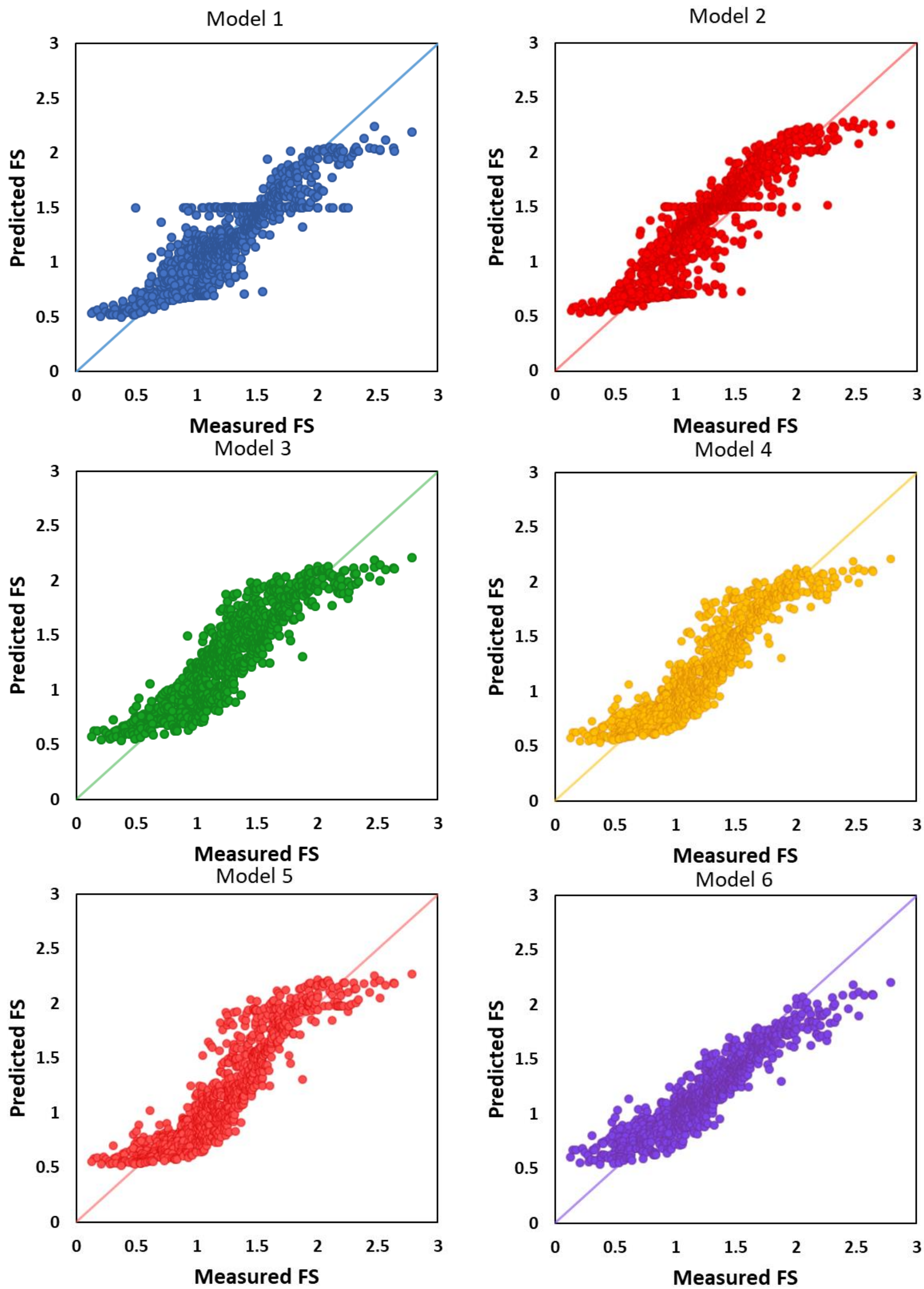


Figure 7. Comparison of Estimated and Simulated Factor of Safety in Different Models

$$\mathrm{RMSE} = \left[\frac{\sum_{i=1}^{n}(S_i - M_i)^2}{n}\right]^{\frac{1}{2}} \frac{100}{\bar{M}} \tag{1}$$

$$\mathrm{CD} = \frac{\sum_{i=1}^{n}(M_i - \bar{M})^2}{\sum_{i=1}^{n}(S_i - \bar{M})^2} \tag{2}$$

$$\mathrm{EF} = \frac{\sum_{i=1}^{n}(M_i - \bar{M})^2 - \sum_{i=1}^{n}(S_i - M_i)^2}{\sum_{i=1}^{n}(M_i - \bar{M})^2} = 1 - \frac{\sum_{i=1}^{n}(S_i - M_i)^2}{\sum_{i=1}^{n}(M_i - \bar{M})^2} \tag{3}$$

$$\mathrm{CRM} = \frac{\sum_{i=1}^{n} M_i - \sum_{i=1}^{n} S_i}{\sum_{i=1}^{n} M_i} \tag{4}$$

$$\mathrm{SE} = \sqrt{\frac{1}{n-2}\left[\sum_{i=1}^{n}(M_i - \bar{M})^2 - \frac{[\sum_{i=1}^{n}(S_i - \bar{S})(M_i - \bar{M})]^2}{\sum_{i=1}^{n}(S_i - \bar{S})}\right]} \tag{5}$$

$$\mathrm{RE} = \left|\frac{M_i - S_i}{M_i}\right| \times 100 \tag{6}$$

$$R = \frac{n(\sum_{i=1}^{n}(P_i)(O_i)) - (\sum_{i=1}^{n} P_i)(\sum_{i=1}^{n} O_i)}{\sqrt{n\left[\sum_{i=1}^{n}(P_i)^2 - \left(\sum_{i=1}^{n}(P_i)\right)^2\right]\left[n\sum_{i=1}^{n}(O_i)^2 - \left(\sum_{i=1}^{n}(O_i)\right)^2\right]}} \;, -1 \le r \le 1 \tag{7}$$

In the aforementioned equations, S_i and M_i denote the simulated and measured values, respectively. $\bar{S}$ and $\bar{M}$ represent the mean of the simulated and measured values, respectively, while n denotes the sample size. Zarei et al. [47] explained the characteristics of these parameters. The minimum value for Mean Error (ME), Root Mean Square Error (RMSE), and Coefficient of Determination (CD) may be equal to zero, while the maximum value for Efficiency Factor (EF) could be one. EF and Correlation Ratio Measure (CRM) may assume values less than zero. A higher RMSE value indicates the accuracy of simulations (whether overestimation or underestimation is present). The CD statistic signifies the relationship between the scatter of simulated values and measurements. EF compares the simulated values with the average of measured values. A negative EF value implies that the average of measured values offers a superior estimate compared to the simulated values. CRM gauges the model's inclination to overfit or underfit the measurements. A negative CRM indicates a propensity to overfit. When all simulated and measured data are identical, the resultant statistics are as follows: ME = 0; RMSE = 0; CD = 1; EF = 1; CRM = 0 [48]. The statistical outcomes derived from the six models are illustrated in Figure 8.

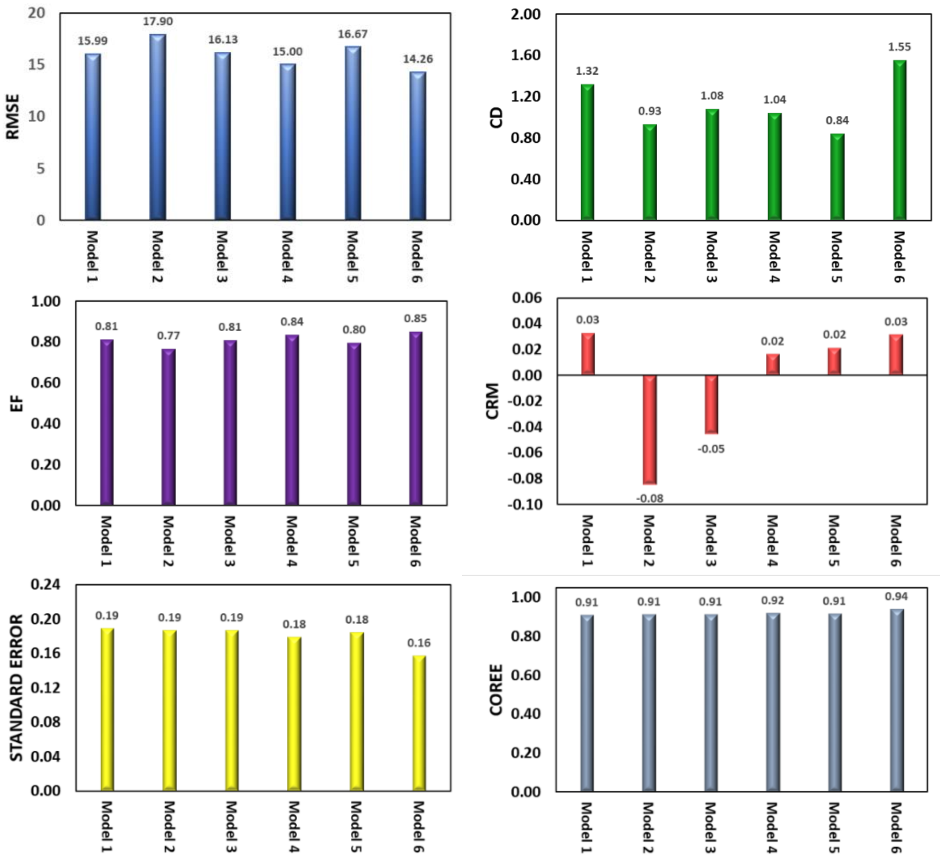


Figure 8. Comparison of statistical parameters for different models.

For a better understanding of each model's behavior, the percentage of data points with relative errors (RE) less than 10, 20, 50, and 100% is counted and shown in Figure 9. Essentially, the higher the percentage of data points with lower REs, the better the model. For example, RE less than 10% indicates an error of less than 10%. The higher the number of data points with errors less than 10% in a model, the more practical and accurate that model is.

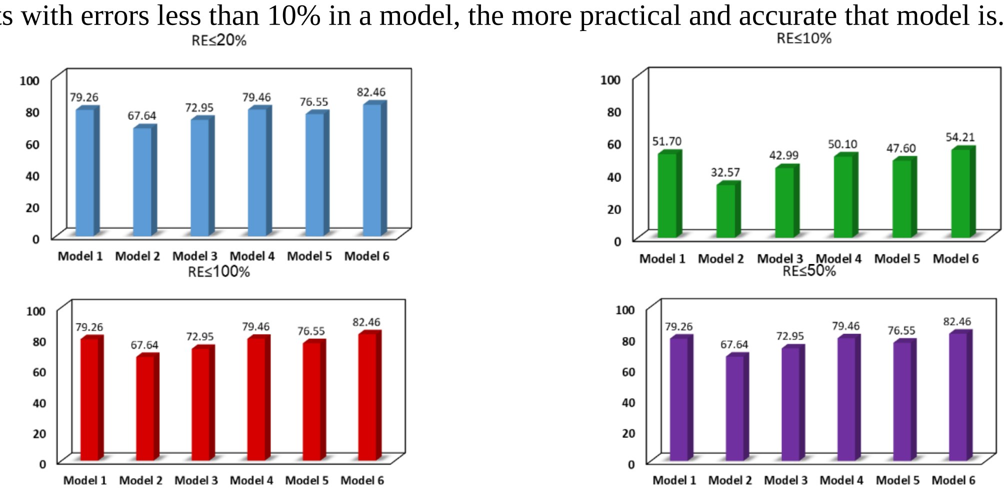


Figure 9. Comparative plots of parameters with RE less than a certain value (%) for different models

In Table 7, all statistical parameters along with their comparison are provided side by side. The best models, in order, are Model 6, Model 4, Model 3, Model 5, Model 1, and Model 2.

Table 7. Comparison of all statistical parameters obtained from the conducted analyses

Parameters	RMSE	CD	EF	CRM	Standard err	COREE	RE≤10%
Model 1	15.99	1.32	0.81	0.03	0.19	0.91	51.70
Model 2	17.90	0.93	0.77	-0.08	0.19	0.91	32.57
Model 3	16.13	1.08	0.81	-0.05	0.19	0.91	42.99
Model 4	15.00	1.04	0.84	0.02	0.18	0.92	50.10
Model 5	16.67	0.84	0.80	0.02	0.18	0.91	47.60
Model 6	14.26	1.55	0.85	0.03	0.16	0.94	54.21

4. Conclusions

Feasibility of the fuzzy method in analyzing the stability of soil slopes was investigated considering the uncertainty of geotechnical parameters. The factor of safety of the slope was evaluated using two methods. The first method involved slope analysis and design in Slide software, where the factors of safety were obtained for 1000 samples using statistical and real data of materials in a soil dam. In this method, three slopes, namely 1:1, 1:2, and 1:2.5, with a constant height of 80 meters were considered, among which slopes with ratios of 1:2 and 1:2.5 were completely stable, while the 1:1 slope exhibited critical conditions. In this scenario, the slope was modeled using probabilistic methods, and the results obtained were extracted as output. The second method aimed at finding the factor of safety in MATLAB software through fuzzy theory, where the slope itself was not modeled. Instead, factors of safety were derived using fuzzy rule definitions and input parameters (i.e., cohesion, internal friction angle, and soil unit weight) to obtain the output results (real factors of safety). Various scenarios were considered for the fuzzy method, many of which were eliminated due to high error rates, and only six scenarios that yielded the best results and were close to each other were selected. Different assumptions were made for these six analyses. Subsequently, statistical equations were employed to compare the six models.

The results of comparing the statistical parameters are summarized in Table 7. Ultimately, by comparing the models, it was concluded that Model 6 (with the assumptions specified for this model in Tables 5 and 6) had the closest values to the real factors of safety and the least error compared to other models, demonstrating better performance. The efficiency of this model is 85%, with a standard error of 16%, which is the lowest error compared to the others. The findings of this study confirm the applicability of the fuzzy method in slope stability analysis. A limitation of this study lies in its exclusive analysis of slopes under dry conditions, without accounting for fluctuations in groundwater levels. Consequently, it is essential to evaluate the method's efficacy across various underground water and drainage scenarios.

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