Efficiency Improvement of Piano Key Weirs Using Numerical Simulation

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ABSTRACT

Water management and conveyance have been crucial aspects throughout the development of human civilization. One of the significant methods of water storage is the construction of dams. Dams require spillways to pass excess water safely. Piano key weirs, as a type of non-linear spillway with specific geometry, have a higher discharge capacity and have thus attracted researchers' attention. The use of piano key weirs is proposed as an alternative solution to improve dam performance. In this research, the performance of piano key weirs was studied using numerical simulations in the Flow 3D software. After evaluating the accuracy of the numerical model results with the experimental results and ensuring the high accuracy of the numerical model, the development and optimization of the numerical model to enhance the spillway's performance in passing flow begins. Based on earlier research, a notable drawback of these spillways is the decrease in the discharge coefficient as the upstream volume flow rate increases, which causes an increase in flow head before the spillway. Therefore, in this research, geometric modifications to the spillway model, including sloping the sidewalls of the spillway by 5, 7, 10 and 12 degrees, were examined. The numerical modeling results indicated an 11% to 15% increase in the discharge capacity of the piano key weir with 5-degree sloped walls, an 8% to 19% increase in the discharge capacity with 7-degree sloped walls, and a 7% to 11% increase in the discharge capacity with 10-degree sloped walls compared to the simple spillway at various upstream volume flow rates. However, in the model constructed with a wall slope of 12 degrees, no increase in the discharge coefficient was observed, and the flow head upstream of the spillway increased.

KEYWORDS

Dam, Piano key weir, Numerical modeling, Discharge coefficient, Flow 3D software.

1. Introduction

Climate change and torrential rainfall worldwide have prompted the International Commission on Large Dams (ICOLD) to issue a statement calling for a review and reassessment of the performance of existing dam spillways. Actions should be taken to rebuild, expand, or strengthen these spillways if needed. According to studies and assessments conducted by the (ICOLD), about 30% of the world's dams face insufficient spillway capacity during floods, which poses a significant and dangerous threat [1]. The mismatch between spillway discharge capacity and incoming floodwaters reduces the spillway's efficiency. As a result, this elevates the burden on the dam. Continuing this cycle can lead to severe problems, including the failure and collapse of dams, causing irreparable damages and costs.

Therefore, given the importance and critical role of spillways, these structures must be robust, reliable, and highly efficient, ensuring they are always at the highest level of readiness to manage excess water and floods. Today, the most advanced type of non-linear spillway is the piano key weir (PKW) 2]. One of the main advantages of piano key weirs is their reduced construction space requirement and smaller dimensions [3]. This results in a reduction in the amount of concrete used and finally reduces costs [4]; in addition, Toulis (2013), in comparing piano key weirs with ogee spillways, has highlighted the economic advantages and lower maintenance costs of piano key weirs. Another significant advantage of piano key weirs is their ability to manage higher flow rates than other spillway types, which can significantly contribute to the dam's safety [5]. The results and analysis by Blancher et al. (2011) demonstrated that the flow capacity of this spillway is at least four times greater than that of a linear spillway (5). Additionally, the unique geometry of piano-key weirs allows them to be installed on the crests of many existing dams, making them one of the most suitable options for upgrading existing spillways 6]. However, optimizing the performance of piano key weirs remains a crucial issue. Researchers have been concentrating on enhancing the discharge coefficient as upstream flow rates increase and minimizing the flow head before the weir. Ribeiro et al. (2009) investigated the impact of parapet walls on the hydraulic efficiency of piano key weirs. These researchers augmented the height of the weir without extending the length of the slopes and obtained that a 12.30% increase in weir height leads to a 15% improvement in its hydraulic efficiency [7]. Anderson (2011), by examining various geometries of piano key weirs and comparing them, concluded that the discharge coefficient of the weir increases as the ratio of the inlet to outlet cycle increases 8]. Machiels et al. (2012) investigated the flow rate through piano key weirs. They concluded that, considering a constant total head on the piano key weir, increasing the slope of the inlet key with the presence of a perimeter wall (parapet wall) results in better performance compared to increasing the slope of the inlet key without the perimeter wall on the weir [9]. Anderson and Tullis (2013), in their study on piano key weirs, discovered that the weir's discharge capacity increases as the ratio of the inlet key width to the outlet key width increases [10]. Falvey (2003) introduced Equation (1), which was previously used for calculating the pass discharge through sharp-crested weirs, as a formula for determining the discharge through piano key weirs [11].

$$Q = C_d \frac{2}{3} L H^{\frac{3}{2}} \sqrt{2g} \quad (1)$$

where Q is the discharge through the weir, Cd represents the weir discharge coefficient, L is the width of the weir, g is the acceleration due to gravity, and H is the flow height over the weir crest. Bremmer and Ortel (2017) demonstrated through numerical simulations that the thickness of the walls of piano key weirs affects the flow discharge coefficient [12]. Al-Baghdadi and Khasaf (2018) examined the impact of varying the crest length of piano key weirs on their discharge capacity. They observed that increasing the crest length significantly affects the discharge capacity of this weir model [13]. Bekheet et al. (2022), through experimental modeling and analysis of the hydraulic behavior of trapezoidal, rectangular, and triangular piano key weirs, concluded that the trapezoidal weir exhibited better hydraulic performance compared to the other shapes, while the triangular weir had the weakest performance. In another part of their study, they used numerical modeling to study the effect of the inlet-to-outlet critical width ratio. Their findings indicated that increasing the inlet-to-outlet critical width ratio to 1.25 resulted in up to a 15% increase in the discharge coefficient [14]. In

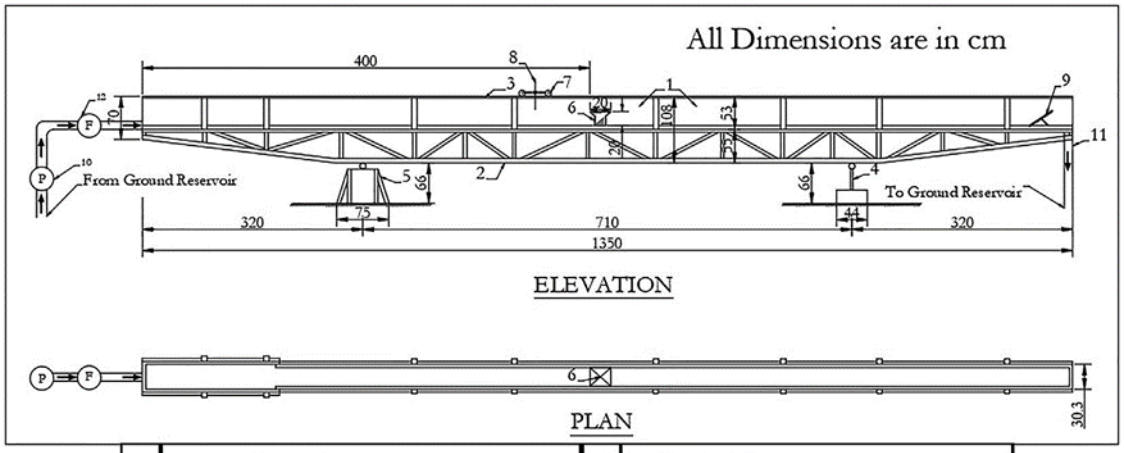
a study by Eltarabily et al. (2025), for different types of piano key weirs, the hydraulic parameters, including water surface profile, flow regimes, and nappes interference, were numerically investigated for different flow rates. The results agreed well, in addition, an empirical equation was proposed for energy dispersion rate prediction in the case of different discharges [15]. Yang et al. (2023) employed numerical simulations to show that geometric modifications can significantly improve PKW discharge efficiency and capacity. These modifications applied the cross-section of each key as a downward semicircle. [16]. Selim et al. (2024) compared PKWs with trapezoidal labyrinth weirs and found that PKWs provide better energy dissipation and discharge performance under free flow conditions [17]. Alizadeh et al. (2023) conducted laboratory tests on A-type triangular PKWs, confirming good discharge capacity and highlighting the potential for further geometric optimization. In their study, the stage–discharge relationship, discharge coefficient, and flow regime were examined [18].

A study by Jin et al. (2025) found that a novel composite Piano Key Weir, featuring both trapezoidal and rectangular sections, dramatically boosts discharge capacity. Numerical tests demonstrated an improvement of up to 16% over conventional trapezoidal designs [19].

Based on earlier research, a notable drawback of these spillways is the decrease in the discharge coefficient as the upstream volume flow rate increases, which causes an increase in flow head before the spillway. However, the present study aims to enhance the discharge efficiency of piano key weirs through geometric modification, specifically by investigating the effects of inclined sidewalls using a numerical approach. To achieve this, the study begins with a comprehensive review of previous research to gather relevant information and select a reliable experimental model for validation. Numerical simulations based on computational fluid dynamics (CFD) are then conducted to model the flow behavior over piano key weirs, with a focus on simulating the spillway geometry and applying the governing flow equations under specified initial and boundary conditions. The numerical model is calibrated and validated using laboratory data to ensure its accuracy. The main novelties of this research include the integration of previous design principles to propose a refined weir geometry and the use of geometric modifications, such as inclined sidewalls, to improve the weir's performance. By enhancing the discharge coefficient and flow capacity, this study provides reasonable insights into improving piano key weir design for greater hydraulic efficiency and better performance under varying flow conditions.

2. Methodology

At first, an appropriate experimental model for developing the numerical model was selected. This case utilizes the experimental model developed by Bekheet et al. (2022) [14]. The experimental setup consisted of an inclined laboratory flume measuring 13.5 meters in length, 0.30 meters in width, and 0.50 meters in depth. The sidewalls were constructed from 10 mm-thick glass panels, reinforced with vertical supporting frames spaced at 1.50-meter intervals. The flume's bed was fabricated from continuous steel sheets spanning its full length. A comprehensive illustration of the flume's structural configuration, including detailed dimensions and design features, is provided in Fig. 1.



1	Flume side walls	7	Rolling carriage
2	Supporting steel truss	8	Point gauge
3	Two rails for the needle gauge	9	Tailgate
4	Tilting support	10	Pump
5	The other support	11	Outlet Pipe
6	PKW position	12	Flowmeter

Figure 1. Layout, details, and dimensions of the flume [14]

In the present research, the rectangular piano key weir model is considered to be in alignment with the research objectives. Fig. 2 shows the rectangular piano key weir experimental model [14]. The geometric specifications of a rectangular weir are provided in Fig. 3. Moreover, Table 1 represents the dimensions of the rectangular piano key weir model considered in this study.

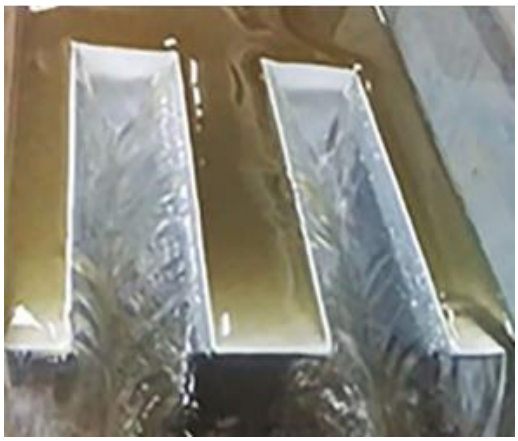


Figure 2. Rectangular PKW model in laboratory [14]

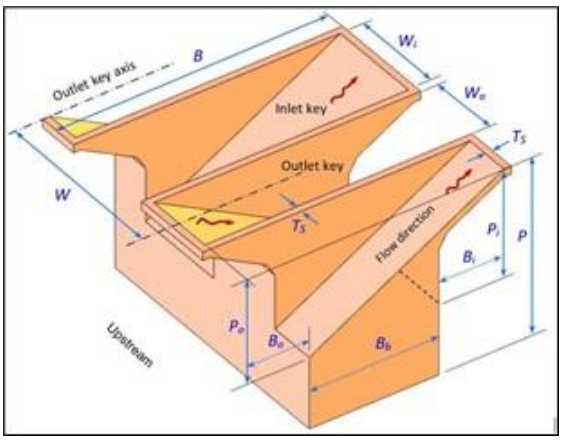


Figure 3. Geometric specifications of PKW [16]

Table 1. The dimensions of the PKW model used in the experimental study (cm) [14].

PKW model's shape	W	L	P	W_i	W_o	B_i=B_o	B	T_s
Rectangular	30.3	151.5	20	6.5	8.25	7.575	30.3	0.2

Subsequently, numerical modeling of the flow over the rectangular piano key weir was performed using Flow-3D software, a powerful tool for numerical simulation and computational fluid dynamics (CFD) [20]. One crucial aspect of numerical modeling is the time required to achieve stable conditions. This study employed a simulation duration of 15 seconds for all models under different conditions and characteristics. After this period, the piano key weir model's flow conditions were stabilized, and the flow characteristics remained consistent. Additionally, the physical conditions for modeling the flow over the piano key weir include a gravitational acceleration of 9.81 m/s2, and the fluid is considered Newtonian. The standard k–ε turbulence model was employed to simulate turbulent flow behavior. A variable time-step size was used in the simulations, automatically adjusted to ensure the Courant number remained within an acceptable range throughout the simulation. In the end, the time-step size was stabilized at 0.15 seconds. Pressure–velocity coupling was handled using the PISO (Pressure Implicit with Splitting of Operators) algorithm, which is an iterative method used to solve the Navier Stokes equations in the numerical modeling. The PISO algorithm improves the stability and accuracy of the solution by performing multiple correction steps for pressure and velocity fields within each time step. At the beginning of the modeling process, a mesh with 400,000 cells specifically for the weir geometry and a 0.01-meter grid size for the weir channel was created (Fig. 6). The optimal mesh dimensions will be determined as the simulation progresses. The PKW geometry was designed using AutoCAD software and exported in the stereolithographic (STL) format. The STL file was then directly imported into Flow-3D for further analysis. Fig. 4 shows the rectangular piano key weir model created in Flow-3D.

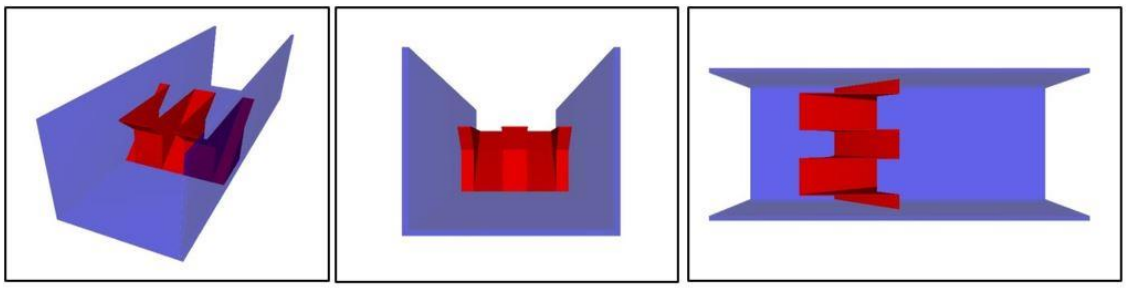


Figure 4. The PKW numerical model created in Flow 3D

3. Results and Discussion

In order to validate the numerical model, a flume replicating the experimental setup used in the laboratory study by Bekheet et al. (2022) was modeled in Flow-3D. The rectangular piano-key weir geometry was constructed based on the exact specifications of the physical experimental model. Boundary conditions matching the experimental setup were applied to the numerical model, including an inlet with a flow rate of 38.89 m^3/h and a flow surface height of 0.225 meters. The flow surface height over the weir crest was then analyzed and quantitatively compared with the experimental results. Figure 1 represents the boundary conditions applied to the numerical model, and the mesh system is shown in Error! Reference source not found.. The boundary conditions have been applied as wall (W), symmetry (S), inlet flow discharge (Q), and outflow (O) to the model.

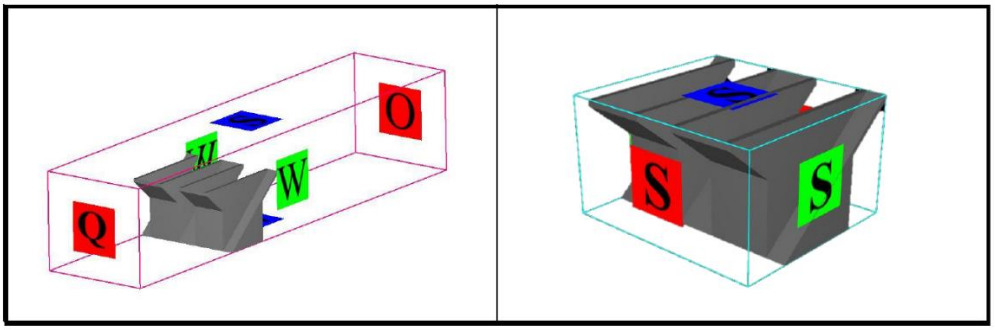


Figure 1. The boundary conditions applied to the numerical model.

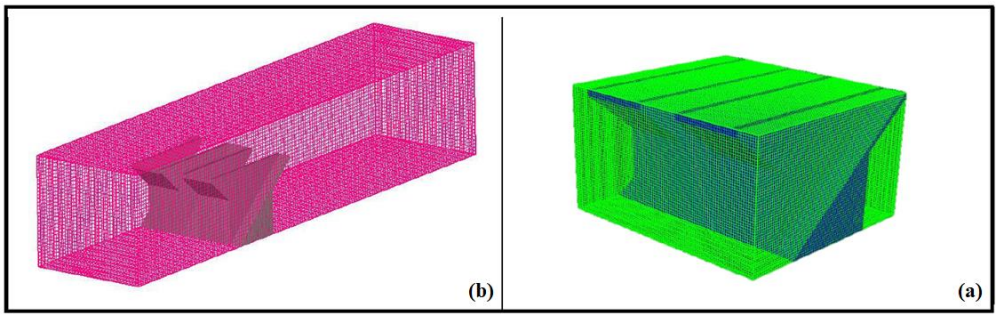


Figure 6. The mesh system applied to the model: (a) weir geometry, (b) the channel.

The model was run in three stages with different mesh sizes including 0.5 cm, 1.0 cm, and 1.5 cm.

Table 1 to Table 3 show the comparison of the results from the experimental model and the numerical modeling. In these Tables, (X_o) represents the distance from the beginning of the weir, (X) denotes the distance from the start of the channel, (B) is the side width of the weir, and (Y) indicates the flow head. Error represents the calculated absolute relative error between the numerical results and the experimental data.

Table 1. Comparison of the experimental data and the numerical results (Mesh size = 0.5 cm)

Mesh 0.5cm	X_o(m)	X(m)	X/B	Y_o(m)	Y(m)	Y/P	Error
Numerical Result	0	0.37425	0	0.02531	0.22531	0.12657	0.14%
Experimental Data			0	0.025	0.225	0.125	
Numerical Result	0.0606	0.43485	0.2	0.02113	0.22113	0.10563	0.30%
Experimental Data			0.2	0.0218	0.2218	0.109	
Numerical Result	0.1212	0.49545	0.4	0.02183	0.22183	0.10915	0.08%
Experimental Data			0.4	0.022	0.222	0.11	
Numerical Result	0.1818	0.55605	0.6	0.02139	0.22139	0.10694	0.28%
Experimental Data			0.6	0.022	0.222	0.11	
Numerical Result	0.2424	0.61665	0.8	0.02157	0.22157	0.10783	0.11%
Experimental Data			0.8	0.0218	0.2218	0.109	
Numerical Result	0.303	0.67725	1	0.01791	0.21791	0.08955	0.42%
Experimental Data			1	0.017	0.217	0.085	

Table 2. Comparison of the experimental data and the numerical results (Mesh size = 1 cm)

Mesh 1cm	X_o(m)	X(m)	X/B	Y_o(m)	Y(m)	Y/P	Error
Numerical Result	0	0.37425	0	0.02742	0.22742	0.13710	1.08%
Experimental Data			0	0.025	0.225	0.125	
Numerical Result	0.0606	0.43485	0.2	0.02372	0.22372	0.11860	0.87%
Experimental Data			0.2	0.0218	0.2218	0.109	
Numerical Result	0.1212	0.49545	0.4	0.02373	0.22373	0.11863	0.78%
Experimental Data			0.4	0.022	0.222	0.11	
Numerical Result	0.1818	0.55605	0.6	0.02341	0.22341	0.11703	0.63%
Experimental Data			0.6	0.022	0.222	0.11	
Numerical Result	0.2424	0.61665	0.8	0.02204	0.22204	0.11020	0.11%
Experimental Data			0.8	0.0218	0.2218	0.109	
Numerical Result	0.303	0.67725	1	0.02068	0.22068	0.10338	1.69%
Experimental Data			1	0.017	0.217	0.085	

Table 3. Comparison of the experimental data and the numerical results (Mesh size = 1.5 cm)

Mesh 1.5 cm	X_o(m)	X(m)	X/B	Y_o(m)	Y(m)	Y/P	Error
Numerical Result	0	0.37425	0	0.04391	0.243912	0.21956	8.41%
Experimental Data			0	0.025	0.225	0.125	
Numerical Result	0.0606	0.43485	0.2	0.02558	0.225584	0.127921	1.71%
Experimental Data			0.2	0.0218	0.2218	0.109	
Numerical Result	0.1212	0.49545	0.4	0.00686	0.206862	0.034308	6.82%
Experimental Data			0.4	0.022	0.222	0.11	
Numerical Result	0.1818	0.55605	0.6	0.00912	0.209122	0.045611	5.80%
Experimental Data			0.6	0.022	0.222	0.11	
Numerical Result	0.2424	0.61665	0.8	0.0086	0.208604	0.043022	5.95%
Experimental Data			0.8	0.0218	0.2218	0.109	
Numerical Result	0.303	0.67725	1	1E-06	0.200001	5E-06	7.83%
Experimental Data			1	0.017	0.217	0.085	

The model with a 0.5 cm mesh size, the small dimensions of cells, and their exact alignment with the weir model's meshing led to highly accurate numerical results compared to the experimental model. As a result, the error between the two models was at most 0.42%. However, this fine mesh size considerably increased the computation time, extending it to 36 hours, which is a notable drawback.

The numerical results for the 1.0 cm mesh model showed acceptable accuracy relative to the experimental data, with a maximum error of 1.69% between the two. Additionally, this mesh size significantly reduced computational time compared to the 0.5 cm mesh model.

The larger cell dimensions and mesh size in the model with a 1.5 cm mesh size resulted in a relatively higher error rate between the numerical and experimental models. Based on the review and evaluation

of the results from the numerical models and considering the primary objectives of the modeling, the model with a 1.0 cm mesh size was selected as the reference model, and subsequent calculations were performed with this mesh size.

Unlike numerous previous studies that validated their numerical models at only a single point on the weir crest, this study conducted validation at six locations along the crest. Using a 5 mm mesh, the maximum error was only 0.42%, indicating good precision. Even with a 10 mm mesh, the error remained low at 1.69%. In contrast, the reference study (Bekheet et al. 2022) used for validation reported a 6.77% error at the upstream point of the weir under similar conditions. The present model, at the same location and with a 10 mm mesh, achieved a much lower error of 1.69%, demonstrating the reasonable accuracy of the current numerical approach.

3.1. Evaluation of the reference model at various upstream flow rates

This study aims to improve the hydraulic efficiency of the piano key weir, particularly by increasing the discharge coefficient. Therefore, the performance of the reference weir model was evaluated at various upstream flow rates. The initial upstream flow rate of 38.89 cubic meters per hour, used for model validation, was incrementally increased by 25% and then 40%, leading to new flow rates of 48.612 and 54.446 cubic meters per hour, respectively. Fig. 7 presents a general overview of the analyzed models and their corresponding names.

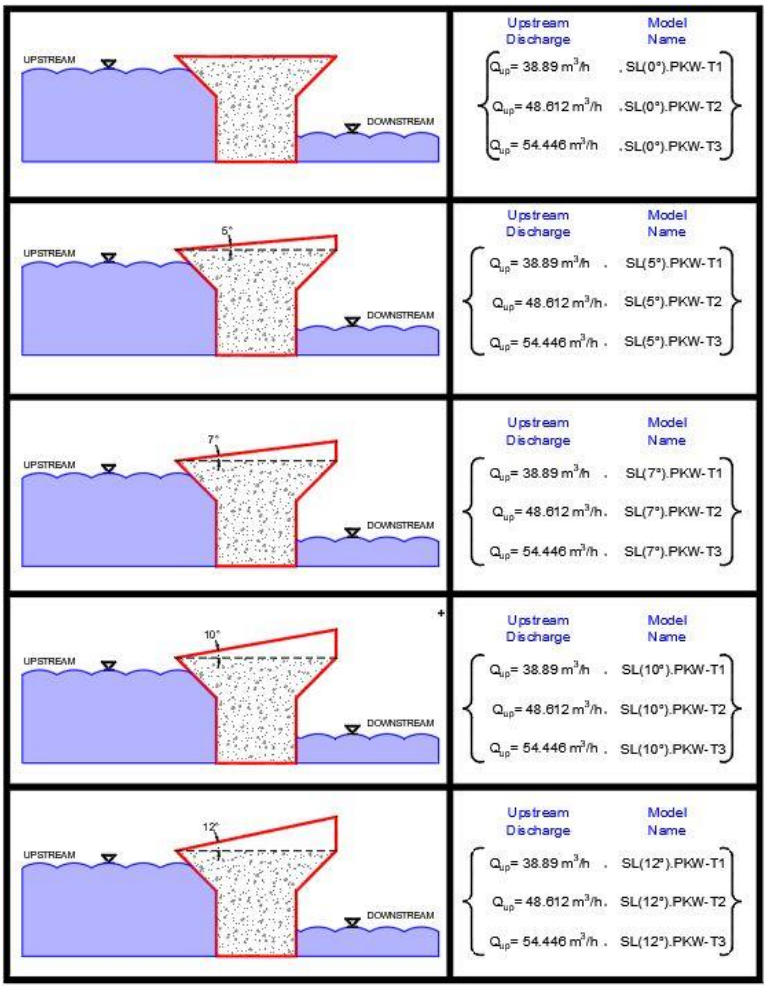


Figure 7. General overview of the analyzed models and their corresponding names.

Table 4 details the numerical results, which outline the models' performance with increasing upstream flow rate to the weir. Figure 2 shows each model's discharge coefficient plotted against H/P. Additionally two charts showing flow discharge through the weir and free surface profile have been presented in Figure 3 and Figure 4 to further illustrate and substantiate the analysis.

Table 4. Comparison of the data from the analysis of the three PKW models at different flow rates

Model name	(H/P)	Weir height (P)	Flow head on weir (H)	Discharge (Q)	Discharge coefficient (C_d)
(SL(0°).PKW-T1)	0.137875	0.2 m	0.027575 m	0.0095 m^3/s	2.1091
(SL(0°).PKW-T2)	0.153825	0.2 m	0.030765 m	0.012090 m^3/s	2.2649
(SL(0°).PKW-T3)	0.167255	0.2 m	0.033451 m	0.01372 m^3/s	2.2664

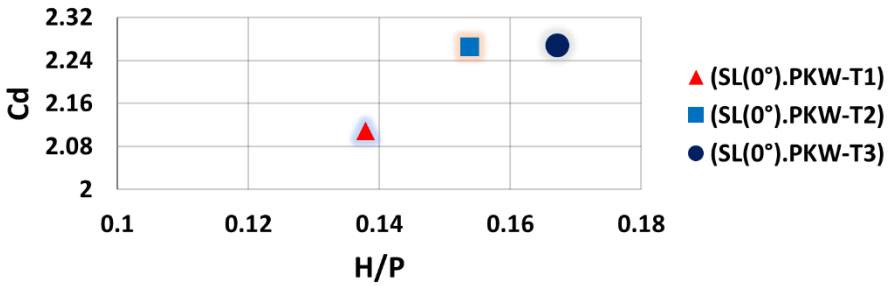


Figure 2. Discharge coefficient plotted against H/P for each of the three models

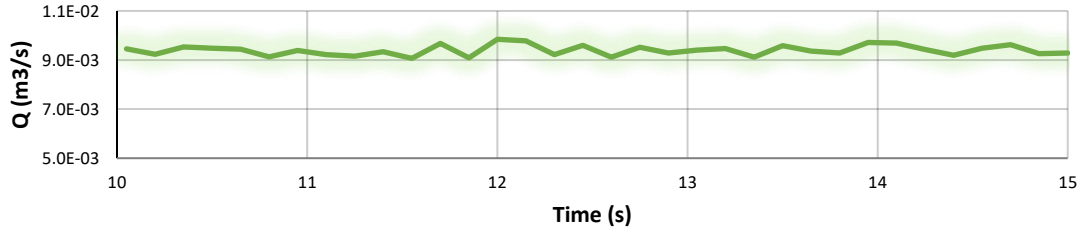


Figure 3. Flow discharge through the weir - (SL(0°).PKW-T1)

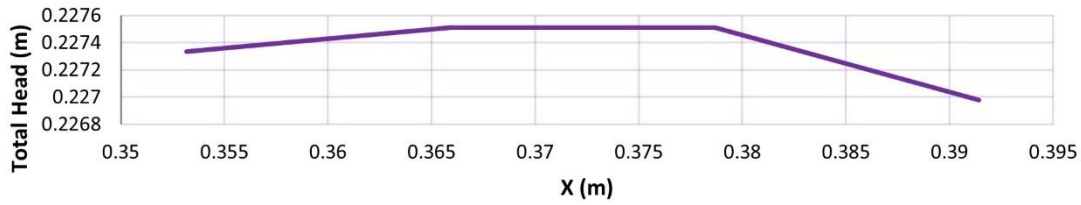


Figure 4. Free surface profile (head) of the flow upstream of the weir - (SL(0°).PKW-T1)

Based on the results and the chart presented in Table 5 and Fig. 8, it is observed that with increasing upstream flow rate, the discharge coefficient in the model (SL(0°).PKW-T2) has increased compared to the model (SL(0°).PKW-T1), and the flow head before the weir has also risen. In the model (SL(0°).PKW-T3), despite the increase in upstream flow rate to the weir and the rise in upstream flow head, the discharge coefficient has not changed compared to the model (SL(0°).PKW-T2). This indicates that with higher flow rates, this particular geometry and model of the weir cannot pass more flow, and the flow head before the weir will continue to increase.

The results indicate that the simple piano

key weir with flat walls experiences reduced discharge efficiency as the upstream flow rate increases and leading to a rise in the flow head before the weir. To enhance the performance of these weirs under high flow conditions, the design of a modified weir with wall slopes of 5, 7, 10 and 12 degrees relative to the horizontal was evaluated and analyzed.

3.2. Evaluation of the performance of the PKW with a 5-degree wall slope at various upstream flow rates

Table 6 presents the numerical results, illustrating the models' performance under increasing upstream flow rates at the weir. Fig. 11 shows each model's discharge coefficient plotted against H/P. Additionally two charts showing flow discharge through the weir and free surface profile have been presented in Fig. 12 and Fig. 13 to further illustrate and substantiate the analysis.

Table 5. Comparison of the data from the analysis of the three PKW models at different flow rates.

Model name	(H/P)	Weir height (P)	Flow head on weir (H)	Discharge (Q)	Discharge coefficient (C_d)
(SL(5°).PKW-T1)	0.08224	0.2128 m	0.0175 m	0.01098 m^3/s	4.7942
(SL(5°).PKW-T2)	0.06824	0.2128 m	0.014521 m	0.01342 m^3/s	7.7532
(SL(5°).PKW-T3)	0.066114	0.2128 m	0.014069 m	0.01537 m^3/s	9.3126

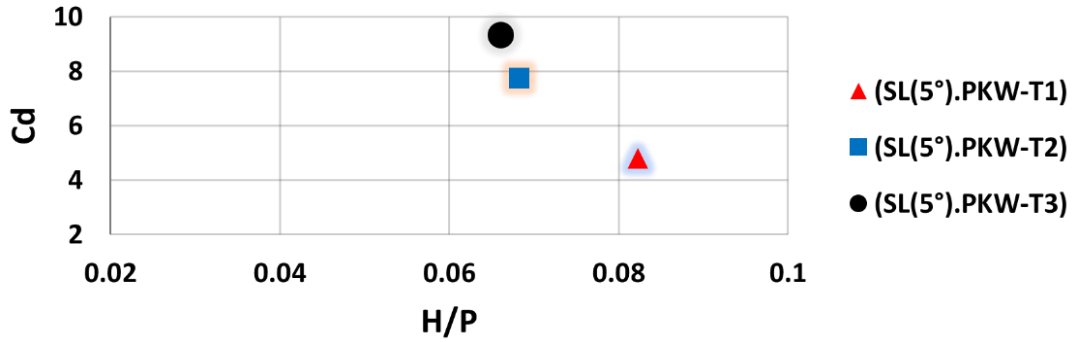


Figure 5. Discharge coefficient plotted against H/P for each of the three models

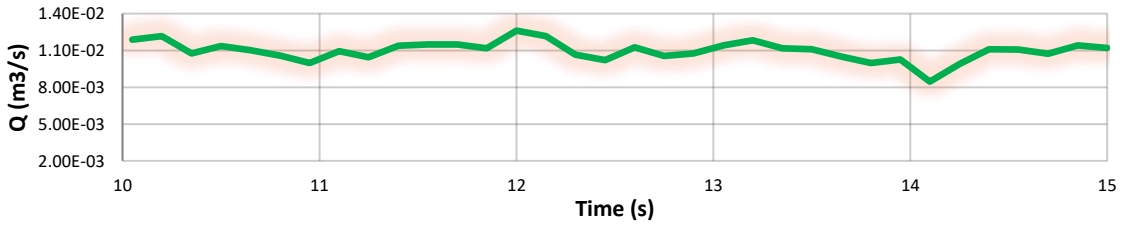


Figure 6. Flow discharge through the weir - (SL(5°).PKW-T1)

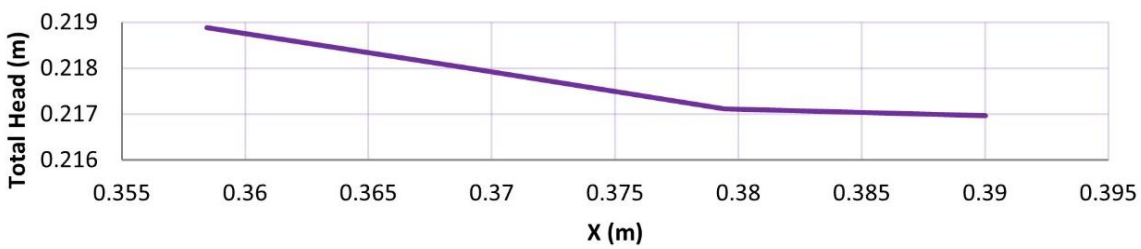


Figure 7. Free surface profile (head) of the flow upstream of the weir - (SL(5°).PKW-T1)

The results and data indicate that the weir with a 5-degree wall slope allows a significantly higher flow rate passed as the upstream inflow increases. The weir's discharge coefficient rises in line with the increasing volume of incoming flow. Additionally, the flow head before the weir decreases as the discharge coefficient improves.

3.3. Evaluation of the performance of the PKW with a 7-degree wall slope at various upstream flow rates

Table 6 presents the numerical results, illustrating the models' performance under increasing upstream flow rates at the weir. Figure 8 shows each model's discharge coefficient plotted against H/P. Additionally two charts showing flow discharge through the weir and free surface profile have been presented in Figure 9 and Figure 10 to further illustrate and substantiate the analysis.

Table 6. Comparison of the data from the analysis of the three PKW models at different flow rates.

Model name	(H/P)	Weir height (P)	Flow head on weir (H)	Discharge (Q)	Discharge coefficient (C_d)
(SL(7°).PKW-T1)	0.09541	0.218 m	0.0208 m	0.01033 m^3/s	3.4802
(SL(7°).PKW-T2)	0.09334	0.218 m	0.02035 m	0.01366 m^3/s	4.7581
(SL(7°).PKW-T3)	0.08738	0.218 m	0.01905 m	0.01626 m^3/s	6.2507

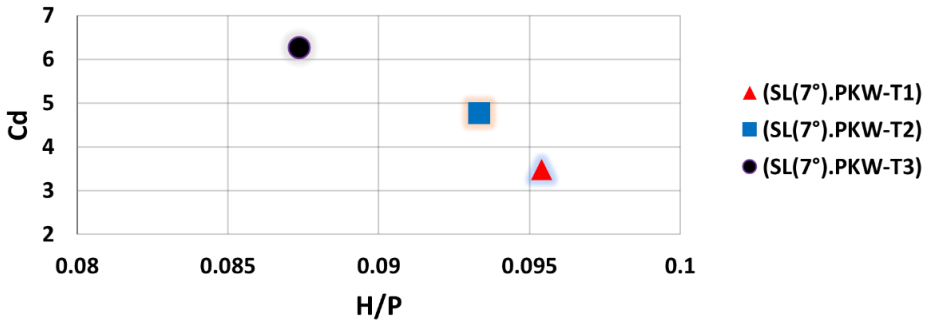


Figure 8. Discharge coefficient plotted against H/P for each of the three models

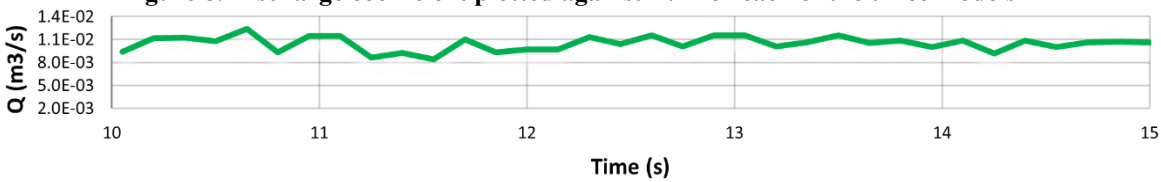


Figure 9. Flow discharge through the weir - (SL(7°).PKW-T1)

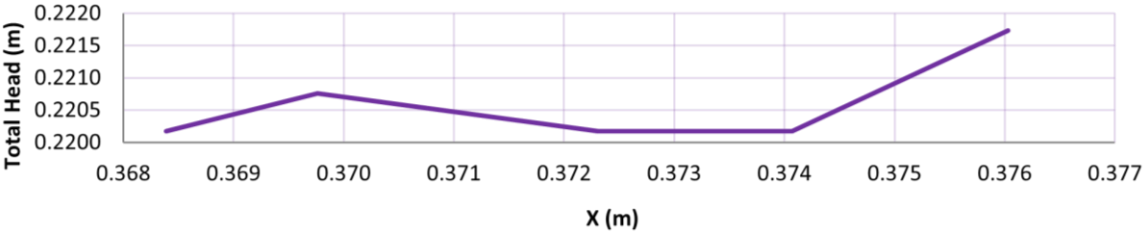


Figure 10. Free surface profile (head) of the flow upstream of the weir - (SL(7°).PKW-T1)

The results and data indicate that the weir with a 7-degree wall slope allows a significantly higher flow rate passed as the upstream inflow increases. The weir's discharge coefficient rises in line with the increasing volume of incoming flow. Additionally, the flow head before the weir decreases as the discharge coefficient improves.

3.4. Evaluation of the performance of the PKW with a 10-degree wall slope at various upstream flow rates

Table 7 presents the numerical results, illustrating the models' performance under increasing upstream flow rates at the weir. Figure 11 shows each model's discharge coefficient plotted against H/P. Additionally two charts showing flow discharge through the weir and free surface profile have been presented in Figure 12 and Figure 13 to further illustrate and substantiate the analysis.

Table 7. Comparison of the data from the analysis of the three PKW models at different flow rates.

Model name	(H/P)	Weir height (P)	Flow head on weir (H)	Discharge (Q)	Discharge coefficient (C_d)
(SL(10°).PKW-T1)	0.136816	0.2264 m	0.031 m	0.01051 m^3/s	1.94451
(SL(10°).PKW-T2)	0.14687	0.2264 m	0.0333 m	0.01297 m^3/s	2.15799
(SL(10°).PKW-T3)	0.059932	0.2264 m	0.01359 m	0.01522 m^3/s	9.71876

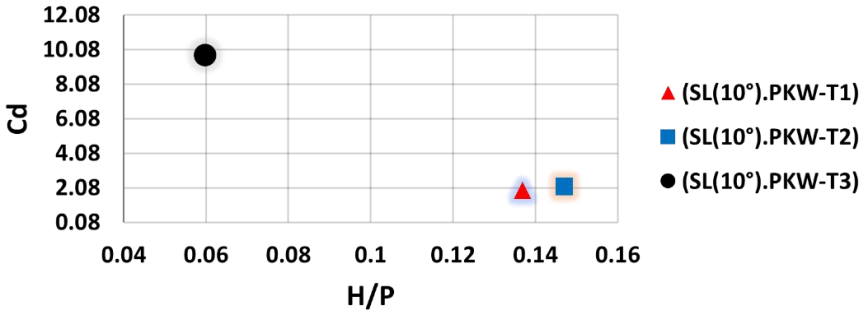


Figure 11. Discharge coefficient plotted against H/P for each of the three models

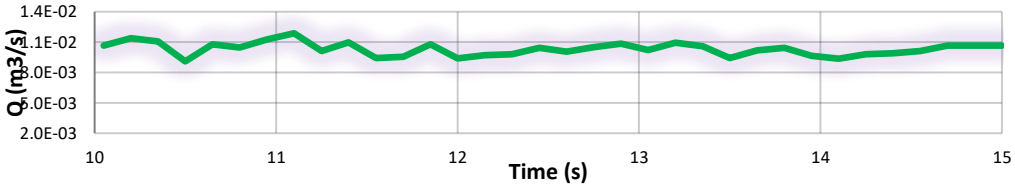


Figure 12. Flow discharge through the weir - (SL(10°).PKW-T1)

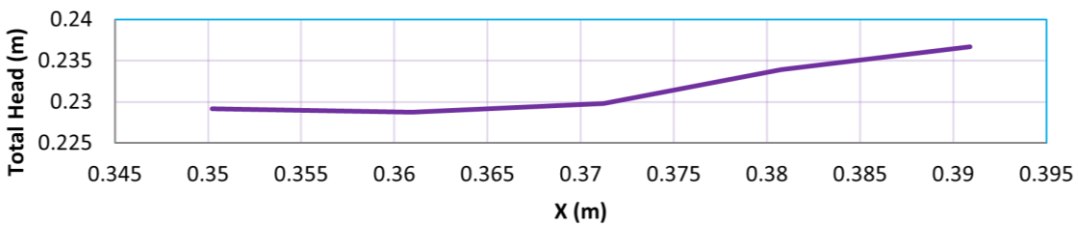


Figure 13. Free surface profile (head) of the flow upstream of the weir - (SL(10°).PKW-T1)

In the piano key weir with a 10-degree wall slope, the results show that while increasing the upstream flow rate from 38.89 to 48.612 cubic meters per hour (25% increase). This only leads to an 11% rise in the discharge coefficient. However, a 40% increase in inflow causes a significant improvement in the discharge coefficient as the weir begins to discharge the flow effectively. This results in a noticeable reduction in the flow head before the weir.

3.5. Evaluation of the performance of the PKW with a 12-degree wall slope at various upstream flow rates

Table 8 presents the numerical results, illustrating the models' performance under increasing upstream flow rates at the weir. Figure 14 shows each model's discharge coefficient plotted against H/P. Additionally two charts showing flow discharge through the weir and free surface profile have been presented in Figure 15 and Figure 16 to further illustrate and substantiate the analysis.

Table 8. Comparison of the data from the analysis of the three PKW models at different flow rates.

Model name	(H/P)	Weir height (P)	Flow head on weir (H)	Discharge (Q)	Discharge coefficient (C_d)
(SL(12°).PKW-T1)	0.0954	0.2311 m	0.043 m	0.01043 m^3/s	1.181
(SL(12°).PKW-T2)	0.2076	0.2311 m	0.048 m	0.01319 m^3/s	1.2683
(SL(12°).PKW-T3)	0.2228	0.2311 m	0.0515 m	0.01469 m^3/s	1.271

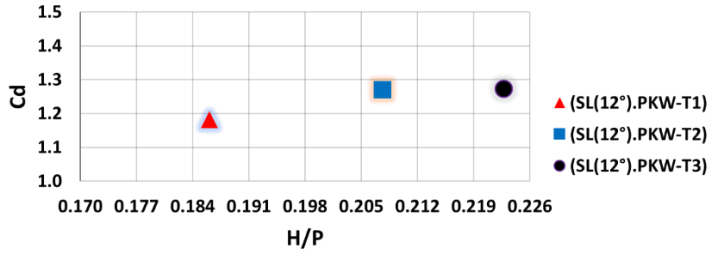


Figure 14. Discharge coefficient plotted against H/P for each of the three models

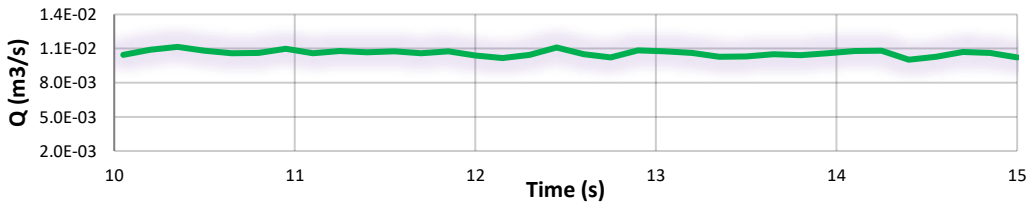


Figure 15. Flow discharge through the weir - (SL(12°).PKW-T1)

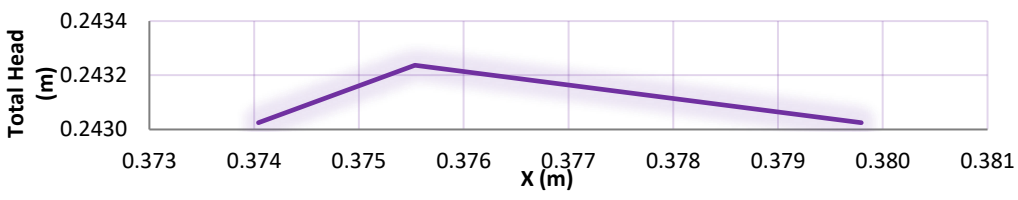


Figure 16. Free surface profile (head) of the flow upstream of the weir - (SL(12°).PKW-T1)

In the piano key weir with a 12-degree wall slope, as indicated by the results, there is no significant increase in the discharge coefficient as the upstream flow rate to the weir increases from 38.89 to 48.612 cubic meters per hour. Additionally, as the upstream flow rate to the weir increases, this geometry is unable to pass the additional flow, leading to an increase in the upstream head of the flow before the weir.
Finally, the results of the discharge coefficients from the numerical simulations for all models were presented in Figure 17 and
Table 9. According to the presented results, the improved models, including weirs with sloping sidewalls, demonstrated a significant increase in discharge rate compared to simple piano key weirs with flat sidewalls. Furthermore, the modified geometries exhibited a marked improvement in the discharge coefficient.

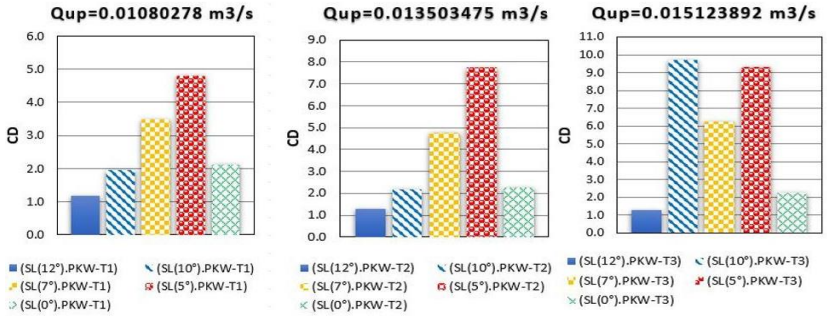


Figure 17. Comparison of the discharge coefficients of the analyzed models.

Table 9. Comparison of the increase in flow discharge rate across the analyzed weirs.

Upstream Volume of Flow Rate = 38.89 m3/hr					
Model Name	**Flow Discharge Through Weir**	**Increase in Flow Discharge Rate**	**Model Name**	**Flow Discharge Through Weir**	**Increase in Flow Discharge Rate**
(SL(0°).PKW-T1)	34.39	15%	(SL(0°).PKW-T1)	34.39	10%
(SL(5°).PKW-T1)	39.52		(SL(10°).PKW-T1)	37.826	
(SL(0°).PKW-T1)	34.39	8%	(SL(0°).PKW-T1)	34.39	9%
(SL(7°).PKW-T1)	37.18		(SL(12°).PKW-T1)	37.534	
Upstream Volume of Flow Rate = 48.612 m3/hr					
Model Name	**Flow Discharge Through Weir**	**Increase in Flow Discharge Rate**	**Model Name**	**Flow Discharge Through Weir**	**Increase in Flow Discharge Rate**
(SL(0°).PKW-T2)	43.525	11%	(SL(0°).PKW-T2)	43.525	7%
(SL(5°).PKW-T2)	48.315		(SL(10°).PKW-T2)	46.693	
(SL(0°).PKW-T2)	43.525	13%	(SL(0°).PKW-T2)	43.525	9%
(SL(7°).PKW-T2)	49.2		(SL(12°).PKW-T2)	47.5	
Upstream Volume of Flow Rate = 54.446 m3/hr					
Model Name	**Flow Discharge Through Weir**	**Increase in Flow Discharge Rate**	**Model Name**	**Flow Discharge Through Weir**	**Increase in Flow Discharge Rate**
(SL(0°).PKW-T3)	49.38	12%	(SL(0°).PKW-T3)	49.38	11%
(SL(5°).PKW-T3)	55.343		(SL(10°).PKW-T3)	54.812	
(SL(0°).PKW-T3)	49.38	19%	(SL(0°).PKW-T3)	49.38	7%
(SL(7°).PKW-T3)	58.53		(SL(12°).PKW-T3)	52.9	

4. Conclusion

The present study investigates the effects of sidewall slope variation on the discharge capacity of Piano Key Weirs (PKWs) through numerical modeling. The numerical simulation results demonstrated acceptable accuracy of the model compared to experimental data. The baseline PKW configuration, with a 0° sidewall slope, was analyzed in three stages under different flow rates to determine its discharge capacity. Subsequently, the weir's discharge performance was compared to the baseline model after modifying the sidewall slope to 5°, 7°, 10°, and 12°.

In the simple piano key weir with a 0-degree wall slope, it was observed that increasing the upstream flow rate decreases its discharge efficiency and leads to a rise in the flow head before the weir. To enhance the performance of the piano key weir, this study simulated and analyzed four different weir geometries with wall slopes of 5, 7, 10 and 12 degrees relative to the horizontal. By comparing the discharge coefficients of these models at identical upstream flow rates, a significant increase in the discharge coefficient was observed for the weirs with sloped walls.

As demonstrated by the numerical model results, the simple piano key weir exhibits a decrease in flow discharge performance as the upstream flow rate increases, leading to a rise in flow head before the weir and a corresponding reduction in the discharge coefficient. However, a significant increase in the discharge coefficient and flow rate can be achieved by sloping the weir's sidewalls. This improvement is due to the increased effective crest length and the stepwise discharge mechanism created by the sloped walls. This geometric modification can also be easily applied to existing simple piano key weirs.

Statements and Declarations:

Author Contributions

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by all authors. The first draft of the manuscript was written by the first author (Behnam Taheri), and all authors commented on previous versions of the manuscript (Ehsan Fadaei-Kermani, Mahnaz Ghaeini-Hessaroeyeh). The final revisions have been applied by Ehsan Fadaei-Kermani and Mahnaz Ghaeini-Hessaroeyeh. Moreover, all authors have read and approved the final manuscript.

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Data Availability

The paper models and data can be available from the corresponding author upon request.

Competing Interests

The authors have no relevant financial or non-financial interests to disclose.

Conflict of Interest

There are no conflicts of interest related to this work.

Consent for Publication

The authors declare their consent to publication of the manuscript by the "journal of Hydraulic Structures".

Consent to Participate

The authors give their consent to take part in this project.

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