Uncertainty analysis for stability and cracking in a gravity dam using fuzzy theory (Case study of Pine Flat Dam)

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ABSTRACT

This study employs fuzzy set theory and the α-cut method to quantify the uncertainty associated with the stability of the Pine Flat concrete gravity dam in California, using Cadam software. Triangular fuzzy numbers were assigned to key input parameters, including dam geometry and loading conditions. Three α-cuts (0, 0.5, and 1), representing 10%, 5%, and 0% uncertainty levels, were used to analyze the impact of input variability on output parameters. Results showed that crack length at the dam heel exhibited the highest sensitivity, with tensile stress at the same location identified as a critical output. A 10% input uncertainty led to a 270% increase in tensile stress uncertainty, highlighting the effectiveness of fuzzy analysis in capturing the nonlinear amplification of uncertainty. These findings underscore the value of fuzzy approaches in dam safety evaluations.

KEYWORDS

Cadam; Crack; Fuzzy; Graviy Dam; Uncertainty.

1. Introduction

Ensuring the safety and structural integrity of concrete dams is of paramount importance, given the potentially catastrophic consequences of dam failure for downstream populations [1]. Concrete dams are subject to various failure mechanisms, most notably sliding, overturning, and the development of tensile cracks or shear/compression-induced collapse [2]. These failure modes are influenced by multiple uncertainties, which can broadly be categorized as hydrological, hydraulic, economic, and structural [3].

Extensive research has been conducted to quantify and analyze these uncertainties. Notably, the application of the Monte Carlo method has been widely adopted in gravity dam analysis. Kostov et al. [4] assessed dam reliability by simulating various loading scenarios and illustrating outcomes using cumulative distribution function (CDF) diagrams. Yanmaz and Bézier [5] applied Monte Carlo simulation to evaluate the stability of Turkey's Porsuk Dam, identifying seven input variables and presenting the probability of failure based on safety factors. Similarly, Kittapcigil [6] examined the reliability of the Turkish Cinder Dam under diverse loading conditions. Altarajos et al. [7] analyzed sliding failure uncertainty using Monte Carlo simulation, showing graphical results of dam failure risk in relation to ten input variables.

Baghlani et al. [8] conducted a comprehensive uncertainty analysis on Bluestone Dam using Monte Carlo simulation integrated with Cadam software, presenting the outcomes through probability density functions. Ghafouri and Moradikia [3] investigated uncertainty and sensitivity at Pine Flat Dam using Cadam and Monte Carlo simulations, finding that a 10% variation in input parameters could lead to a 128% change in the dam's maximum tensile stress—emphasizing the significant influence of upstream water level over sediment pressure and seismic loading.

Further contributions to uncertainty modeling include the work of Ghorbani et al. [9], who used probabilistic analysis to assess how concrete strength variability affects scaled accelerograms in dynamic dam analysis. Mohammadi et al. [10] explored flood-related reservoir uncertainty using Monte Carlo simulation, revealing minimal variations in water levels of Dez, Karun, and Gotvand dams—0.037%, 0.107%, and 0.034%, respectively—caused by fluctuating inflow hydrographs.

In terms of cracking and failure mechanisms, several studies have investigated the physical behavior of dams under stress. Lin et al. [22] analyzed crack propagation in arch dams based on fracture toughness, using the Xiluodu Dam as a case study. Their findings established a correlation between field deformation monitoring data and crack propagation. Rebeiro and Leger [23] reviewed numerical modeling approaches and material models used to evaluate the safety of cracked concrete dams, applying Abaqus and the concrete damage plasticity model to simulate cracking behavior.

Liu et al. [24] examined seismic tensile cracking and its influence on slope stability in cemented sand and gravel (CSG) dams.

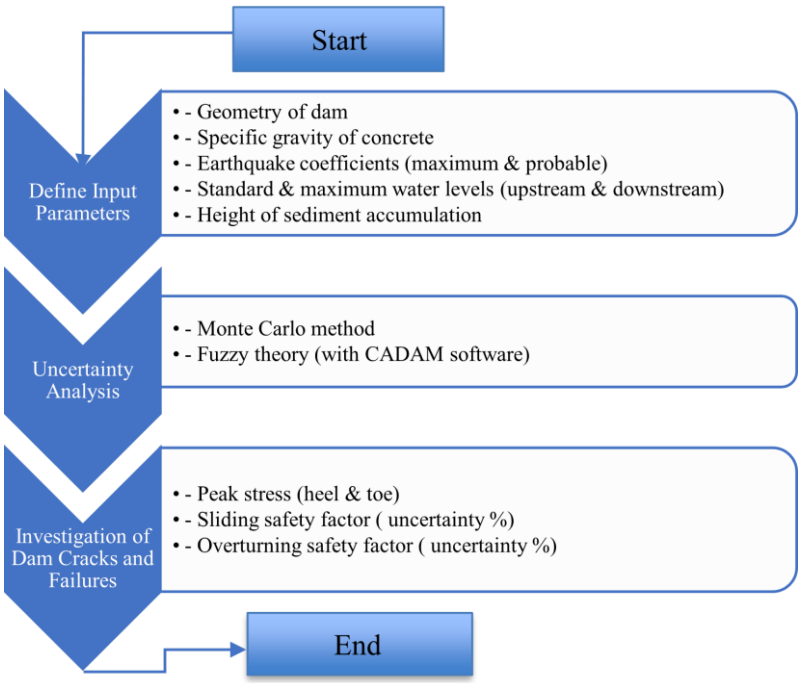


Figure. 1. Summary of the research conducted in this study.

Min et al. [25] showed that surface cracking in upstream zones may induce hydraulic fracturing, presenting a critical crack depth of 12.93 m and a width of 1.01 mm—2.53 times greater than the allowable limit. Zhou et al. [26] investigated the delayed failure of Brazil's Feijão Dam, proposing a mechanism involving progressive slip surface growth along weak layers. Xu et al. [27] proposed a three-dimensional branch-shaped failure mechanism and a finite element limit equilibrium method for analyzing rock dam stability, revealing that increased dam height and slope shift the critical slip surface to shallower regions.

In this context, the present study applies fuzzy set theory and the α-cut approach to evaluate the uncertainty in output parameters of the Pine Flat concrete gravity dam in California. Cadam software is used to model structural parameters and applied loads, with triangular fuzzy membership functions defining input uncertainty. Three α-cut levels (0, 0.5, and 1) are used to represent uncertainty levels of 10%, 5%, and 0%, respectively. The focus is on output parameters including safety factors against sliding and overturning, maximum tensile stresses at critical zones (heel and toe), and crack formation at the dam–foundation interface. The analysis particularly emphasizes tensile stress at the heel, which reduces the effective contact area and promotes crack initiation.

Overall, this research demonstrates how the fuzzy approach effectively captures nonlinear propagation of uncertainty in dam safety analysis, providing insights that complement and, in some respects, surpass traditional probabilistic methods [28].

2. Case Study

This research investigated the geometry and construction materials of the Pine Flat gravity concrete dam. Situated on the Kings River, approximately thirty miles east of Fresno, California, USA, the dam was primarily designed for flood control and to provide irrigation water for agriculture. The reservoir associated with the dam offers a diverse range of recreational facilities for both visitors and travelers. The dam's crest spans 560 m and consists of 37 blocks, each with a width of 15.2 m. The analysis included a detailed examination of the maximum block height, which measures 121.92 m. Fig.2, 3, and 4 show the cross-sectional view of the dam [3], the the structural view of the Pine Flat Dam, and the dam location map, respectively.

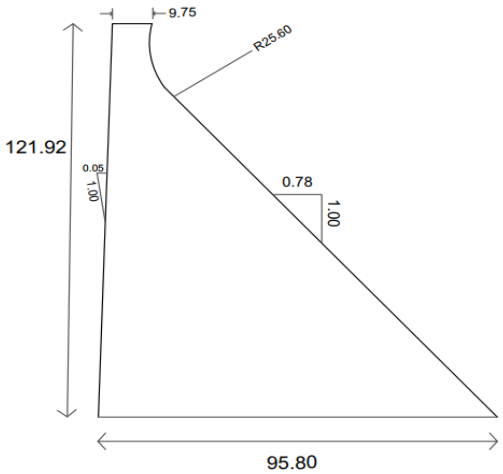


Figure 2. Geometry of Pine Flat Dam (m) [29].

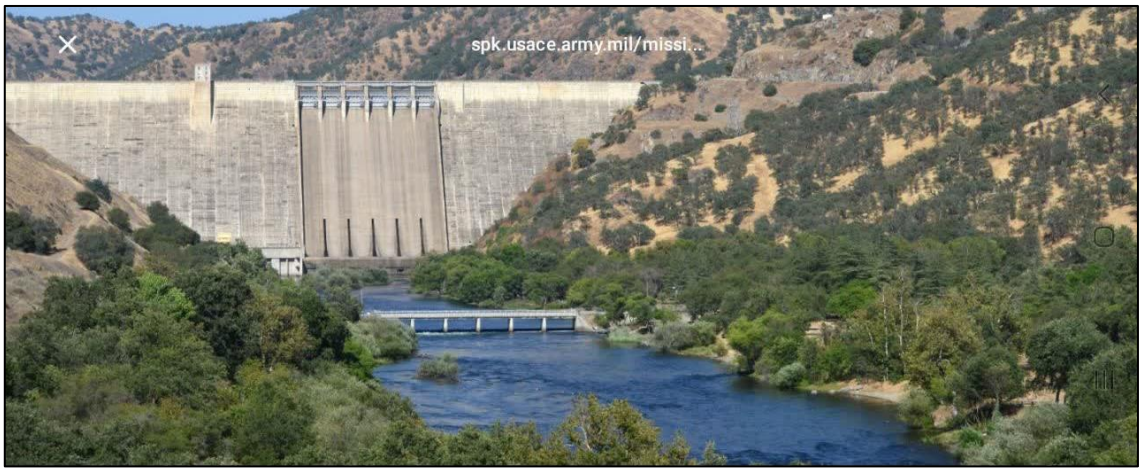


Figure 3. View of Pine Flat Dam Structure across Kings River.

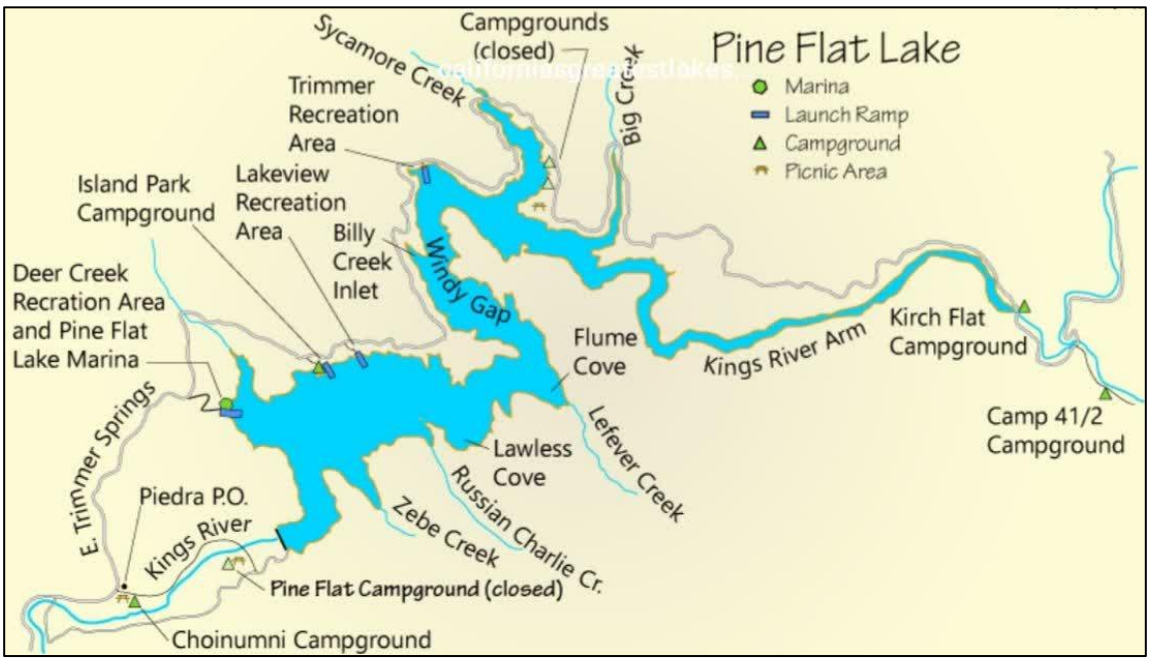


Figure 4. Map of Pine Flat Dam Location in California.

3. Material and Methods

The analysis of dam stability focuses exclusively on load combinations with a high likelihood of occurrence [2]. Table 1 outlines the standardized load combinations established by the United States Army Corps of Engineers (USACE), which include usual, unusual, and extreme load conditions.

Table 1. load conditions used for the stability analysis of gravity dams [31].

	Class	Earthquake	Headwater	Tail Water	Silt	Ice	Uplift
LC1	Unusual	-	-	-	-	-	-
LC2	Usual	-	Crest (top of gate)	Minimum	Yes	Yes	Yes
LC3	Unusual	-	SPF	Flood elevation	Yes	-	Yes
LC4	Extreme	OBE U.D.	-	-	-	-	-
LC5	Unusual	OBE D.D.	Usual pool level	Minimum	Yes	-	P.E.L.
LC6	Extreme	MCE D.D.	Usual pool level	Minimum	Yes	-	P.E.L.
LC7	Extreme	-	PMF	Flood elevation	Yes	-	Yes

Note: SPF (Standard Project Flood), PMF (Probable Maximum Flood), OBE (Operating Basis Earthquake), MCE (Maximum Credible Earthquake), U.D. (Upstream Direction), D.D. (Downstream Direction), P.E.L. (Pre-Earthquake Level), LC (Load Condition).

In Table 1, The load combinations (LC1 to LC7) include various classes of loading conditions categorized as Usual, Unusual, or Extreme, reflecting the severity and expected frequency of each scenario. Earthquake inputs include terms such as OBE (Operating Basis Earthquake) and MCE (Maximum Considered Earthquake), with directional indicators like U.D. (Upstream Direction) and D.D. (Downstream Direction) specifying the orientation of seismic forces. Headwater conditions are defined by levels such as the usual pool level, SPF (Standard Project Flood), PMF (Probable Maximum Flood), and crest elevation (top of gate). Tail water levels refer to either minimum downstream water elevation or flood elevation under extreme conditions. The presence of silt and ice loads on the upstream face of the dam are marked where applicable. Uplift pressures at the dam base are also considered, with some cases referring to P.E.L. (Potential Extreme Level), indicating uplift corresponding to critical high-water scenarios.

The gravity dam must adhere to specific criteria to resist both overturning and sliding. The following equations provide a method for calculating the safety factors against sliding and overturning. Eq. [1] defines the computation of the sliding safety factor for a horizontal surface under static conditions.

$$F_{sliding} = \frac{Total\ resistive\ forces\ along\ the\ slip\ plane}{Total\ driving\ forces\ along\ the\ slip\ plane} = \frac{\sum F_R}{\sum F_O} \quad (1)$$

Eq. [2] defines the computation of the overturning safety factor, which is determined by dividing the cumulative resisting moments by the total overturning moments relative to the toe or heel of the dam [2].

$$F_{overturning} = \frac{Total\ resistant\ moments}{Total\ driving\ moments} = \frac{\sum M_R}{\sum M_O} \quad (2)$$

The USACE has defined stability parameters for overturning and sliding, in addition to imposing constraints on critical internal stresses. Table 2 presents the safety factors for stability and sliding, as well as the stress criteria applicable to gravity dams under various loading conditions.

Table 2. USACE stability and stress criteria for gravity concrete dams [30].

Loading Condition	Sliding Safety Factor	Overturning Safety Factor	Compressive Stress	Tensile Stress
Usual	2	1.3	$0.5f'_c$	0
Unusual	1.7	1.2	$0.3f'_c$	$0.6f'^{\frac{2}{3}}_c$
Extreme	1.3	1.1	$0.9f'_c$	$1.5f'^{\frac{2}{3}}_c$

Note: f'_c is 1-year unconfined compressive strength of concrete.

3.1 Fuzzy Theory

A fuzzy set comprises elements that exhibit non-deterministic properties, each with a specific degree of membership. Unlike deterministic sets, where elements are strictly either members or non-members, fuzzy sets assign membership degrees based on specific criteria. Fuzzy sets can be better understood through a comparison with classical sets and the concept of membership degree. In classical sets, each element has a membership degree of one, while others have a membership degree of zero. An element is distinctly categorized as either a member or a non-member within classical sets. In contrast, fuzzy sets lack precise and clearly defined boundaries [19].

3.1.1 Fuzzy Analysis of Pine Flat Dam

The input parameters of the model in this study are defined as triangular fuzzy functions, incorporating the stresses applied to the dam and the material properties of the dam structure. The base value, which represents the unambiguous parameter value, is considered the most likely value with a membership degree of one in its fuzzy function. The number was recognized as an α-cut with a membership degree of one, along with another α-cut defining the 'abutment' interval as $[x_a, x_b]$, which corresponds to a membership degree of zero. Through the application of these two α-cuts, a triangular fuzzy function was formulated for each input parameter. To determine the minimum and maximum values of the "abutment" interval $[\boldsymbol{x_a}, \boldsymbol{x_b}]$, the desired input parameter was initially evaluated with certainty as the mean of the specified range. Subsequently, it was adjusted by increments and decrements of 10 percent to calculate the corresponding minimum and maximum values of the "abutment" interval. If a fuzzy number, denoted as N, is considered a subset of real numbers. Then every variable, like x, is assigned a membership degree within the interval [0,1] for $x \in N$, denoted as $\mu N(\epsilon)$, which reflects the degree of belonging to the fuzzy number. If x_c denotes the parameter value without uncertainty, it is shown in Eq. [3] and [4] [19].

$$x_{a.a} = x_c - \Delta x_\alpha \quad (3)$$

$$x_{b.a} = x_c + \Delta x_\alpha \quad (4)$$

According to Fig. 5, for the abutment interval [xc+Δx] and [xc-Δx] , Eq. [5] and [6] yield constant values.

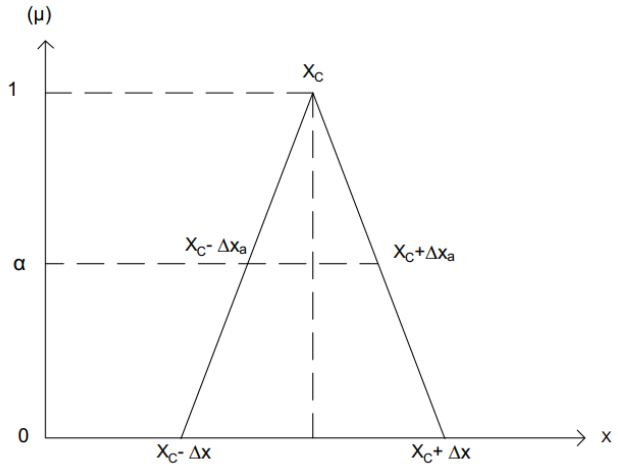


Figure 5. presents the input parameters in fuzzy format [10].

$\Delta x = 0.1x_c$ [5]

$\Delta x = 0.05x_c$ [6]

This entails identifying two α-cuts-one denoting the vertex of the fuzzy triangle set at one, and the other representing its base set at zero. A triangular fuzzy function was constructed for each specific input parameter. The values of xa, xb, and xc in the α-cut abutment, which is set at zero and signifies a 10% uncertainty in the desired input parameters, will be calculated [19]. At the outset, a fuzzy stability analysis and crack investigation of the Pine Flat concrete dam were conducted using deterministic values, devoid of uncertainty. An analysis of the maximum stresses within the dam structure, the stability coefficients related to sliding and overturning, and the extent of cracking at the dam heel-compared to the regulatory limits set by USACE- Table 3 shows that these values are comfortably below the specified thresholds. As a result, the dam is considered stable and compliant with the regulations.

Subsequent to an uncertainty-free assessment of dam stability, a fuzzy analysis was undertaken to account for uncertainty. Unlike conventional analytical methodologies, several geometric and technical parameters of the dam lack definitive and unambiguous values due to inherent uncertainties. Evaluating these uncertain values is imperative, considering the characteristics of the parameters, their significance, and the designer's perspective. Following a rigorous analysis of dam stability, a subsequent fuzzy analysis incorporating uncertainty was conducted. This study established triangular fuzzy functions for the dam material properties, foundation, and model input parameters, including variables such as water pressure on the upstream face of the dam, dam-specific gravity, and earthquake force

Table 3: The dam's stability factors were evaluated assuming no uncertainty.

Loading Condition	Sliding Safety Factor	Overturning Safety Factor	Compress Stress (KPA)	Tensile Stress (KPA)
Usual	2.01	1.82	-797	-1029
Unusual	1.75	1.47	-358	-2767
Extreme	1.35	1.34	-0.38	-2983

Note: fc' is 1-year unconfined compressive strength of concrete. In this case fc'=35MP

acceleration. The base value, defined as crisp value, was determined to be the most probable value with a membership degree of one in its fuzzy function. Tables 4 and 5 present the input parameters used in the Cadam software.

Table 4. Input parameters of the fuzzy model considering 5% uncertainty.

Parameter	Unit	Minimum value of the parameter x_a	Baselineparameter value x_c	Maximum value of the parameter x_b
Specific gravity of concrete	Kg/m^3	2357.9	2482	2606.1
Maximum earthquake coefficient	g	0.095	0.1	0.105
Probable earthquake coefficient	g	0.1425	0.15	0.1575
Standard water level on the upstream side	m	95	100	105
Standard water level on the downstream side	m	14.25	15	15.75
Maximum water elevation on the upstream side	m	104.5	110	115.5
Maximum water level on the downstream side	m	19	20	21
Height of sediment accumulation in the dam reservoir	m	19	25	21

Table 5. Input parameters of the fuzzy model considering 10% uncertainty.

Parameter	Unit	Minimum value of the parameter x_a	Baselineparameter value x_c	Maximum value of the parameter x_b
Specific gravity of concrete	Kg/m^3	2233.8	2482	2730.2
Maximum earthquake coefficient	g	0.09	0.1	0.11
Probable earthquake coefficient	g	0.135	0.15	0.165
Standard water level on the upstream side	m	90	100	110
Standard water level on the downstream side	m	13.5	15	16.5
Maximum water elevation on the upstream side	m	99	110	121
Maximum water level on the downstream side	m	18	20	22
Height of sediment accumulation in the dam reservoir	m	22.5	25	27.5

Fig. 6 demonstrates the need to obtain a corresponding α-cut in the output parameter fuzzy function for each α-cut in the input parameter fuzzy functions. Due to software limitations that accommodate only single values rather than numerical α-cuts, a methodology is required to extract the α-cut of the output fuzzy function without encompassing the entire range of values within the α-cut of the input parameter fuzzy functions. Treating the desired output parameter as a function of the designated fuzzy parameters, the scenario is formulated as an optimization problem. The constraints of this optimization problem are defined by the fuzzy input parameter range within the desired α-cut. To compute the α-cut of the output parameter, determining the maximum and minimum α-cuts from the optimization problem is sufficient.

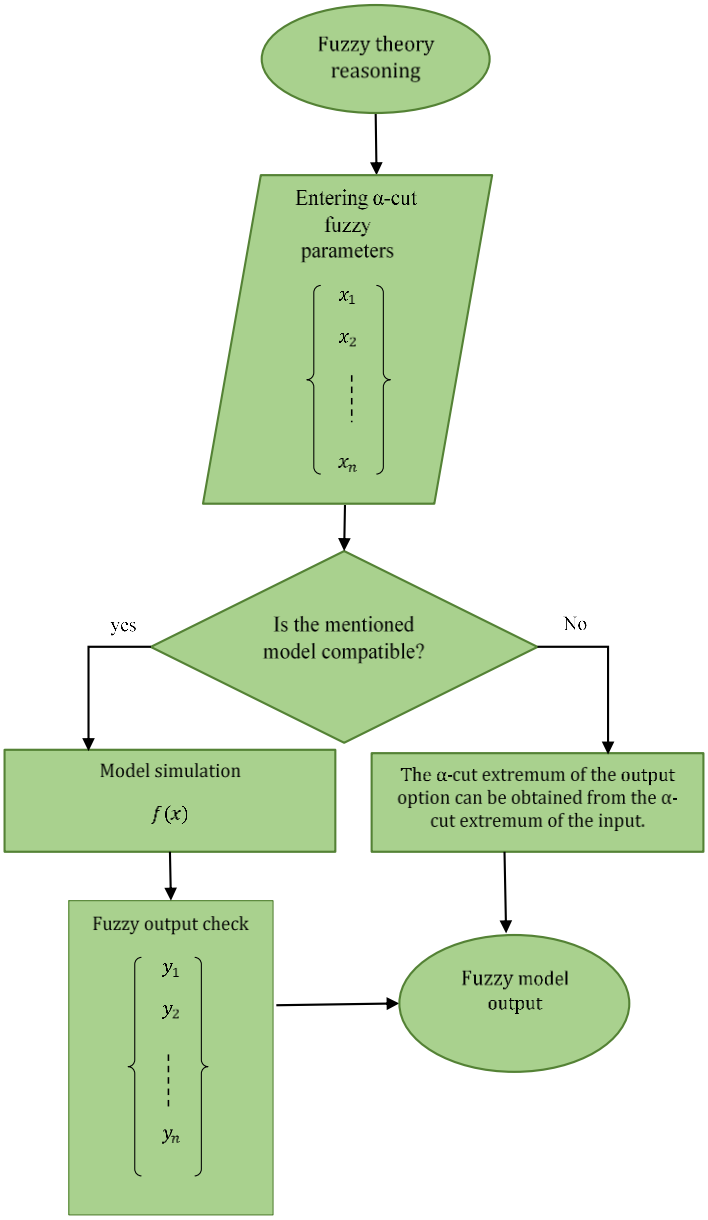


Figure 6. Consistency analysis of the fuzzy model in CADAM software.

In scenarios where the target output parameter exhibits non-monotonic behavior with respect to the parameters defined as fuzzy functions, an optimization strategy is essential to determine the minimum and maximum values of the output parameter's fuzzy function. Conversely, if the function is monotonic, an optimization procedure is unnecessary for determining these values. The minimum and maximum α-cuts of the input parameters play a crucial role in determining the corresponding α-cuts of the output parameter. The study by Ghafouri and Moradikia [3] highlights a novel issue in analyzing fuzzy input parameters for a concrete gravity dam model. Their research demonstrates that extracting the minimum and maximum α-cuts from the fuzzy functions of the input parameters is the primary method.

Consequently, the obtained α-cut serves as a means to define the fuzzy function of the desired output parameter. The decision to integrate the minimum and maximum α-cuts of an input parameter into the model hinges on the parameter's diminishing or augmenting impact on the desired output parameter. When the value of an input parameter rises proportionally with the desired output parameter, it is categorized as an augmenting factor. Similarly, a diminishing parameter can be characterized [31].

The Cadam software employs fuzzy analysis techniques to evaluate overturning and sliding stability coefficients in a concrete dam model. The structural model is created by inputting the dam's geometric dimensions, concrete's specific gravity, friction coefficient, and adhesion value at the dam-foundation interface.

Following this, dam modeling involves inputting the upstream and downstream water heights, specific gravity, sediment height, and gravitational acceleration coefficient to account for seismic forces.

The Cadam software can process various code loading combinations within a single model and present the results of specified parameters through a unified analysis, expediting result acquisition. This capability is particularly beneficial for research, including the current study, which requires extensive analysis. A detailed depiction of the research modeling process is provided in Fig. 7.

To establish the fuzzy functions of the output parameters, the diminishing and augmenting parameters outlined in Table 6 are first identified. After their classification as diminishing or augmenting, the minimum and maximum values for each α-cut of the fuzzy function corresponding to the specified coefficients are calculated.

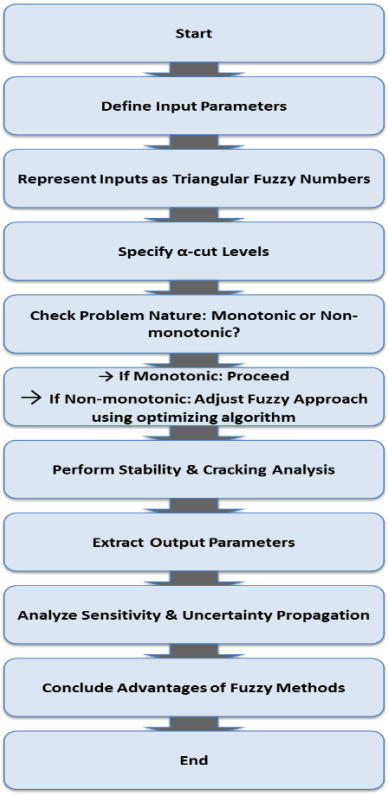


Figure 7. Comprehensive examination of the research methodology.

Table 6. Factors influencing the safety coefficients and stresses on the dam [21].

Input parameter	Safety factor for sliding stability	Safty factor for downstream overturning stability	Maximum tensile stress at the dam heel	Peak compressive stress at the dam toe	Number of safty factor at the dam toe
Water height upstream	Reduce	Reduce	Increase	Increase	Increase
Height of sediment accumulation behind the dam	Reduce	Reduce	Increase	Increase	Increase
Specific gravity of sediments	Reduce	Reduce	Increase	Increase	Increase
Specific gravity of concrete	Increase	Increase	Reduce	Reduce	Reduce
Seismic acceleration coefficient	Reduce	Reduce	Increase	Increase	Increase

Note: In load combination cases 1 and 4, as specified in Table 1, the impact of concrete specific gravity is the inverse of the values presented in Table 5.

4. Results and Discussion

4.1 The Analysis of Crack Values, Stresses, and Confidence Factors

To obtain the required fuzzy functions, three α-cuts (0, 0.5, and 1), representing 10%, 5%, and 0% uncertainty, respectively, were used. Subsequently, the Cadam program generated an output parameter consisting of five analytical models for each fuzzy function.

Tables 7 to 11 present the results of the models developed for all four output alternatives. The conclusions of this study align with the findings reported by Moradikia et al. in reference [21].

Table 7 shows that each variation in the input parameters results in a twofold variation in the output parameters. In summary, a 5% and 10% variability in the input leads to a 10% and 20% variability in the critical stresses. Moreover, in the absence of lateral forces such as seismic activity and hydrostatic pressure, the safety margins against overturning and sliding remain strong, eliminating the need for further investigation. The dam remains crack-free due to the absence of tensile stress. Regarding load combination 1, the sensitivity of the critical tensile and compressive stresses remains consistent.

Table 7. Outcomes for load combination 1.

Output parameter	In the absence of uncertainty	+5% uncertainty	-5% uncertainty	+10% uncertainty	-10% uncertainty
Peak stress at the dam heel(KPA)	-2767	-2906	-2629	-3044	-2490
Uncertainty percentage at the dam heel	-	10.0		20.0	
Peak stress at the dam toe(KPA)	-120	-126	-114	-132	-108
Uncertainty percentage at the dam toe	-	10.0		20.0	
Sliding safety factor	*	*	*	*	*
Percentage of uncertainty in the sliding coefficient of confidence	-	-		-	
Overturning safety factor	*	*	*	*	*
Percentage of uncertainty in the overturning safety factor	-	-		-	
Crack length as a percentage of the dam's length(M)	0	0	0	0	0
Percentage of uncertainty in crack length	0	0		0	

Note: The asterisk (*) in the table indicates an infinite value.

Table 8. out put for load combination 2.

Output parameter	Crisp	+5% uncertainty	-5% uncertainty	+10% uncertainty	-10% uncertainty
Peak stress at the dam heel (KPA)	-797	-1129	-455	-1306	-88
Uncertainty percentage at the dam heel	-	84.6		152.8	
Peak stress at the dam toe (KPA)	-1029	-856	-1187	-753	-1361
Uncertainty percentage at the dam toe	-	32.2		59.1	
Sliding safety factor	2.05	1.45	2.19	1.17	2.50
Percentage of uncertainty in the sliding coefficient of confidence	-	40.2		94.1	
Overturning safety factor	1.82	1.6	2.1	1.41	2.24
Percentage of uncertainty in the overturning confidence factor	-	27.5		45.6	
Crack length as a percentage of the dam's length	0	0	0	0	0
Percentage of uncertainty in crack length	0	0		0	

Table 8 indicates that the most notable sensitivity is associated with the critical tensile stress at the dam's heel, reaching 152%. This highlights the critical nature of tensile stress in load combination 2, as shown in Fig. 8, where the highest principal stress occurs at both the heel and toe of the dam. Furthermore, the results indicate the absence of structural cracks under this specific loading scenario, suggesting that the dam's stress levels remain stable in load combination 2, even with a 10% uncertainty.

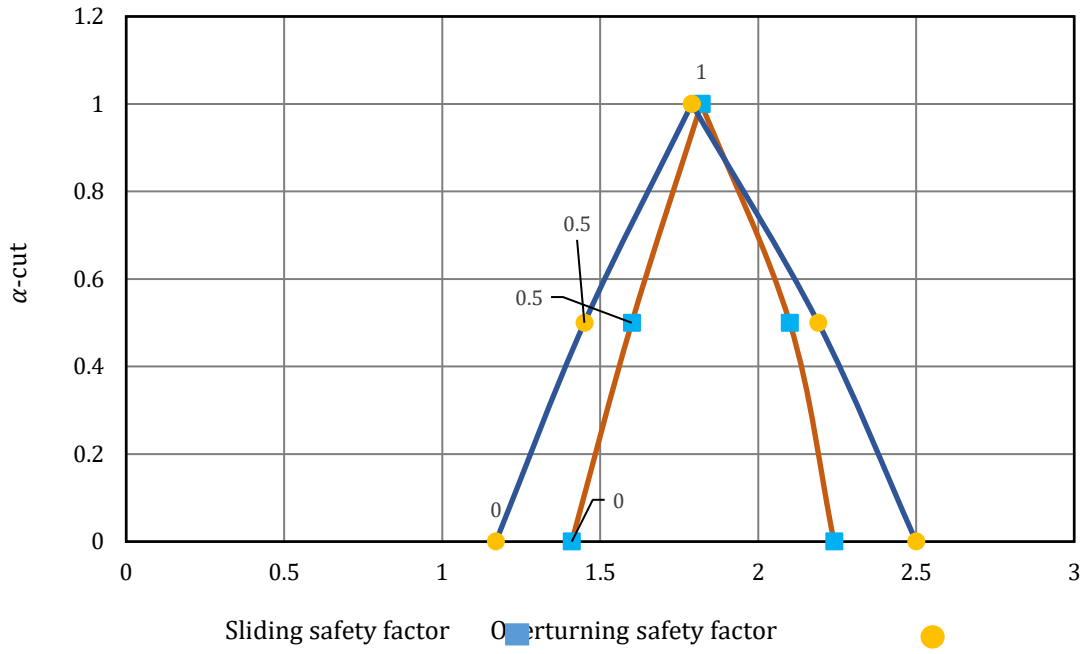


Figure. 8. Fuzzy function of the maximum principal stress at the dam's heel and toe under load combination 2.

Table. 9. Results for load combination 3.

Output parameter	In the absence of uncertainty	+5% uncertainty	-5% uncertainty	+10% uncertainty	-10% uncertainty
Peak stress at the dam heel	-358	-743	0.3	-970	0.38
Uncertainty percentage at the dam heel	-	207.6		270.8	
Peak stress at the dam toe	-1344	-1151	-1562	-974	-2059
Uncertainty percentage at the dam toe	-	30.5		80.7	
Sliding safety factor	1.75	1.06	1.73	0.89	1.98
Percentage of uncertainty in the sliding coefficient of confidence	-	47.7		76.9	
Overturning safety factor	1.54	1.31	1.76	0.89	1.98
Percentage of uncertainty in the overturning confidence factor	-	29.2		70.8	
Crack length as a percentage of the dam's length	0	0	8	0	36
Percentage of uncertainty in crack length	0	-		-	

Table 9 indicates that the tensile stress at the dam's base, with a variation of 270.8%, exhibits the highest degree of uncertainty regarding the stability of Pine Flat Dam, based on the assessment of cracks at the base in load combination 6. This is visually represented in Fig. 9, while Fig. 10 presents the safety factor values for both sliding and overturning. The results from this load combination suggest that, despite a 5% uncertainty in the dam's input parameters, a crack may form at the dam's base. However, this does not necessarily indicate imminent instability, as the modeling in this scenario assumes zero adhesion between the concrete and its foundation. Conversely, the regulatory framework for this load combination accounts for some tensile strength at the concrete-foundation interface, meaning that, in practice, crack formation may be prevented.

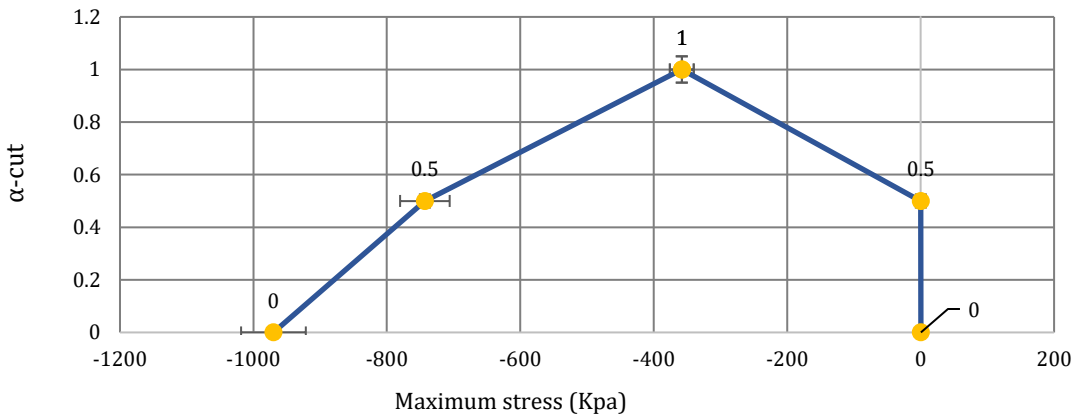


Figure 9. Fuzzy function of the maximum principal stress at the dam's base in load combination 3.

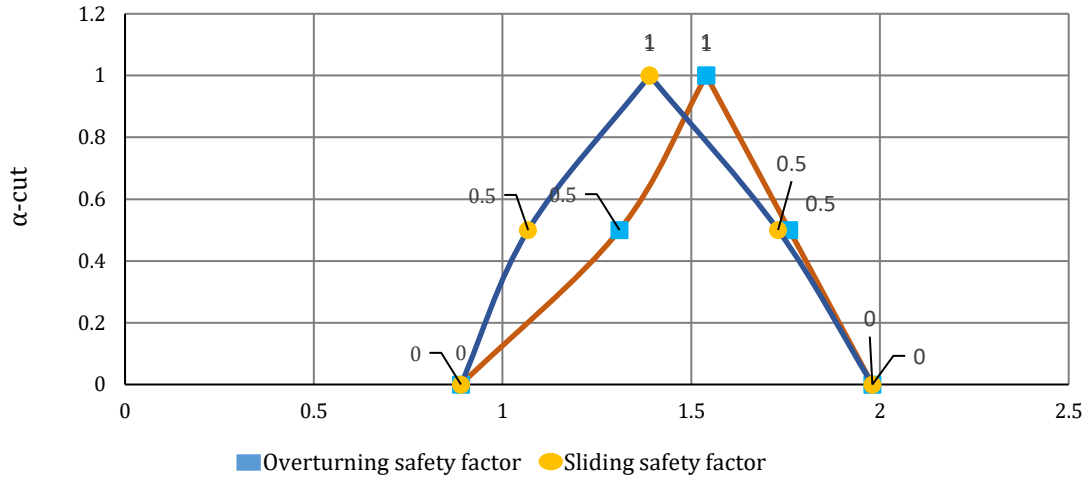


Figure 10. Fuzzy function of the safety factor against sliding and overturning in load combination 3.

Table 10. Results for load combination 4.

Output parameter	In the absence of uncertainty	+5% uncertainty	-5% uncertainty	+10% uncertainty	-10% uncertainty
Peak stress at the dam heel	-3388	-3597	-3020	-3811	-2983
Uncertainty percentage at the dam heel	-	17.0		24.4	
Peak stress at the dam toe	0.44	-1037	0.45	-1123	0.5
Uncertainty percentage at the dam toe	-	23578.1		255340.9	
Sliding safety factor	*	*	*	*	*
Percentage of uncertainty in the sliding coefficient of confidence	-	-		-	
Overturning safety factor	*	*	*	*	*
Percentage of uncertainty in the overturning confidence factor	-	-		-	
Crack length as a percentage of the dam's length	14.7	13.8	15.7	12	16.7
Percentage of uncertainty in crack length	-	12.9		31.9	

Note: The asterisk (*) in the table indicates an infinite value.

Table 10 demonstrates that there is a 5% uncertainty regarding the potential deformation at the dam's toe, which may subsequently lead to the development of cracks at the critical tensile stress point. A 10% discrepancy in the input parameters can introduce a 31.9% uncertainty in crack length. The minimal initial value of the dam's essential tensile stress gives rise to a substantial level of uncertainty, potentially creating a misleading perception.

Table 11. Results for load combination 5.

Output parameter	In the absence of uncertainty	+5% uncertainty	-5% uncertainty	+10% uncertainty	-10% uncertainty
Peak stress at the dam heel	-41	-554	0.41	-652	0.5
Uncertainty percentage at the dam heel	-	1350.2		1589.0	
Peak stress at the dam toe	-1786	-1011	-2082	-551	-3047
Uncertainty percentage at the dam toe	-	59.9		139.8	
Sliding safety factor	1.12	0.93	1.34	0.76	1.51
Percentage of uncertainty in the sliding coefficient of confidence	-	36.9		66.3	
Overturning safety factor	1.47	1.3	1.92	1.15	2.1
Percentage of uncertainty in the overturning confidence factor	-	42.2		64.6	
Crack length as a percentage of the dam's length	0	0	21	0	52.5
Percentage of uncertainty in crack length	-	-		-	

Table 11 demonstrates that a 5% deviation in the input parameters leads to a 21% increase in crack length, while a 10% uncertainty results in a 53% increase, compromising 53% of the dam base and rendering it unsafe. Consequently, load combination 5 poses a significant risk, especially due to the potential exacerbation by seismic forces. In contrast, the water pressure in other cases, such as load combinations 2 and 3, is insufficient to independently induce such a crack. Therefore, the combined effects of elevated water levels and seismic forces substantially increase the likelihood of additional cracks in the dam.

This unexpected result in Table 12 can be explained as follows: the initiation of a fracture at the dam heel causes the dam's base - which rests on the ground and serves to distribute force and generate stress on the foundation - to lift off the ground. The decrease in the foundation's cross-sectional area leads to an exponential increase in tensile stress. The results in Table 12 show an abnormal rise from 1,735 to 7,908, ultimately destabilizing the entire dam. This is illustrated in Fig. 11 for crack size and Fig. 12 for the safety factor against sliding and overturning.

Table 12. Results for load combination 6.

Output parameter	In the absence of uncertainty	+5% uncertainty	-5% uncertainty	+10% uncertainty	-10% uncertainty
Peak stress at the dam heel	0.5	-53	0.55	-324	0.65
Uncertainty percentage at the dam heel	-	10490.0		64670.0	
Peak stress at the dam toe	-2241	-1961	-3020	-1735	-7908
Uncertainty percentage at the dam toe	-	47.0		275.5	
Sliding safety factor	1.35	0.79	1.44	0.65	1.66
Percentage of uncertainty in the sliding coefficient of confidence	-	35.3		64.7	
Overturning safety factor	1.34	1.19	1.5	1.05	1.65
Percentage of uncertainty in the overturning confidence factor	-	23.1		44.8	
Crack length as a percentage of the dam's length	18.5	0	45	0	81
Percentage of uncertainty in crack length	-	243.2		437.8	

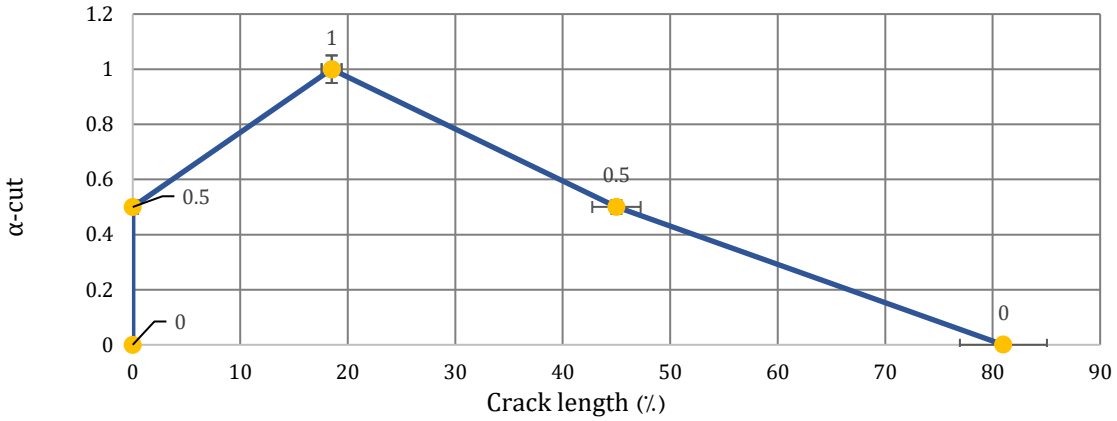


Figure 11. Fuzzy function of the crack amount for load combination 6.

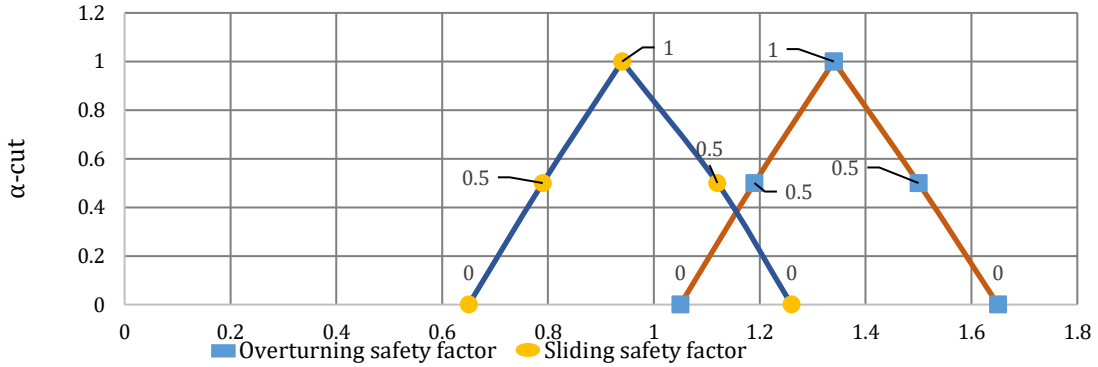


Figure 12. Fuzzy function of the safety factor against sliding and overturning for load combination 6.

5. Results and Discussion

This study presents an analysis of the stability of the Pine Flat gravity dam using fuzzy set theory and the α-cut method. To establish the necessary fuzzy functions, three α-cuts were used: 0, 0.5, and 1, corresponding to 10%, 5%, and 0% uncertainty, respectively. The α-cut at 0 is the most significant, as it represents a 10% uncertainty in the model's input parameters, resulting in the highest uncertainty in the output parameters. Therefore, the results of this α-cut are thoroughly examined. Nevertheless, the percentage uncertainties in the output parameters were excessively large, particularly in the maximum tensile stresses under loadings 5 and 6. This issue stemmed from the low base parameter values, set at half a kilopascal. Consequently, the percentage uncertainties calculated based on these values are unreliable. By representing model input parameters associated with uncertainty-such as dam material specifications and applied loads-as triangular fuzzy functions and incorporating them into the Cadam concrete dam analysis programs, the safety factors against sliding and overturning, as well as the principal stresses and fissures at the dam's base, were determined as fuzzy functions. An uncertainty analysis of the specified output parameters was then performed using the derived fuzzy functions. Given a consistent geometry for concrete gravity dams, the elastic modulus and specific gravity of the dam can be determined using fuzzy analysis by considering the applied loads and only the minimum and maximum input parameter values. Consequently, fuzzy values for output parameters, such as critical stresses, crack thresholds, and safety factors against sliding and overturning, can be identified without the need for an optimization algorithm.

1) The results of this study suggest that the model's fuzzy responses are more practical than its classical counterparts, providing enhanced insights into the dam's future performance.
2) Introducing minor inaccuracies in the input parameters of the weighted dam analysis leads to significant changes in the output parameters. A 10% deviation in the model's input parameters could result in a 270.8% variation in the maximum tensile stress at the dam toe.
3) The spectrum of cracking variations in concrete gravity dams ranges from minor surface damage to significant structural failure. A 10% variability in the input parameters results in a 437.8% variation in the output parameters associated with the dam's crack. This unforeseen outcome can be explained as follows: the initiation of a crack at the dam's heel leads to the uplifting of the dam's base, which serves as an abutment to distribute force and generate tension on the foundation. Consequently, the reduction in the foundation's cross-sectional area causes a significant rise in tensile stress, increasing from 1,735 to 7,908, as demonstrated in the results table, ultimately destabilizing the entire dam.The stability coefficients for overturning and sliding, along with the stresses and cracks observed in the dam under specific conditions, adhered to regulatory limits, indicating stability. Conversely, under uncertainty, these parameters exceeded regulatory thresholds, leading to the dam's destabilization.

Statements and Declarations

The authors declare that they have no conflict of interest.

Competing Interests

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Authors' Contributions

All authors contributed to the study conception and design. Material preparation, data collection, and analysis were performed by Zahra Ashrafi, Mehdi Komasi, and Nazila Kardan. The first draft of the manuscript was written by Zahra Ashrafi and all authors commented on previous versions of the manuscript.

Data Availability

The data presented in this study are available on request from the corresponding author.

Consent for Publication

All the authors have read and agreed to the submitted version of the manuscript.

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