Risk Analysis of Internal Erosion in Earth Dams Considering Uncertainties in the Permeability Coefficient of Materials

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ABSTRACT

Earth dams play a crucial role in water supply and power generation; therefore, ensuring their stability, safety and identifying various potential failure mechanisms are of great importance.
Among different failure modes, internal erosion (piping) is one of the primary factors contributing to the instability and eventual failure of earth dams. In this study, a probabilistic framework is proposed to assess the risk of internal erosion in earth dams considering uncertainty in the permeability coefficient of materials. The Lar Dam was selected as a case study. Seepage analysis was performed using the finite element method in SEEP/W software under transient conditions based on a 22-year time series of reservoir water levels. The uncertainty in permeability coefficients of dam materials was modeled using normal probability distributions. To quantify the risk of internal erosion, two point-estimate methods, namely Rosenblueth and Harr methods, were applied. The results showed that the probability of internal erosion is 0.22 and 0.21 using Rosenblueth and Harr methods, respectively, with a standard deviation of approximately 0.10. These results indicate a relatively high risk of internal erosion in Lar Dam. Furthermore, the close agreement between the two methods demonstrates that the Harr method can be used as a computationally efficient alternative for probabilistic risk assessment in cases with symmetric distributions. The proposed approach provides a practical framework for incorporating uncertainty into internal erosion risk analysis of earth dam.

KEYWORDS
internal erosion risk, earth dam, uncertainty, permeability coefficient, Harr method, Rosenblueth method.

1. Introduction

Dams, as one of the main engineering structures in water resource management and flood control, play a vital role and hold particular importance in providing drinking water, supporting agriculture, and generating hydroelectric power [1]. Any dam failure can lead to significant human and financial losses. The seriousness of dam failure stems from the severe risks it poses to human communities. Given the importance of dams—especially earth dams, which are more prevalent and widely used—examining and analyzing the factors that lead to their failure is critically important [2]. Because of the key role of earth dams in achieving multiple objectives, the stability of these structures is also of great significance. Earth dams are susceptible to various failure modes, such as overtopping, earthquakes, internal erosion, slope instability, and more. Among these, the risk stemming from internal erosion has been studied occasionally as a critical factor threatening the stability of earth dams. Some of these studies are as follows:

Delavar et al. (2016) investigated internal erosion and seepage in earth dams using the Natural Element Method (NEM) [3]. They first validated the accuracy of NEM by simulating the behavior of porous flow and calculating leakage discharge in a semi-permeable weir, and then applied the method to the Doroodzan Dam. Their results showed that the proposed method, with an average error below 5%, has suitable accuracy for analyzing this phenomenon. They also reported that the absence of internal erosion and piping in the dam body—due to the constancy of the permeability coefficient over time—confirms the proper performance of the body drain. Fattahi et al. (2020) conducted a case study to analyze seepage in the foundation and body of the Roudbar Lorestan earth dam using a numerical finite element method [4]. The results showed that the water seepage through the cross-section of the dam's foundation and body was equivalent to 0.27% of the dam's total water volume. Moreover, the dam demonstrated a very high safety factor against piping. Kalate and Kheiri (2022), taking into account uncertainty in hydraulic conductivity, examined seepage in earth dams using the finite element method and Monte Carlo simulation (MCS) [5]. To predict the Effective Seepage Discharge (ESD), they employed Support Vector Regression (SVR) and Gene Expression Programming (GEP). With a correlation coefficient of 0.96 and a root mean square error of approximately 0.01, the SVR model exhibited higher accuracy than GEP and showed closer agreement with the finite element analysis. Koohdaraq et al. (2024) used SEEP/W software to study seepage flow and hydraulic gradient in the body and foundation of the Alavian earth dam [6]. The purpose of their research was to analyze the effects of seepage flow and hydraulic gradient, as well as to evaluate various seepage control methods such as drains, cutoff walls, and clay blankets. Foster et al. (2000) presented a method for estimating the relative probability of earth dam failures due to piping, based on an analysis of historical dam failures and incidents [7]. They approximated a dam's probability of failure from piping by using weighted factors. Data on the time interval of piping and its warning signs indicate that piping usually develops rapidly, providing little time for remedial measures. However, in certain cases, the piping process progresses slowly enough to allow sufficient time for reservoir drawdown or other corrective actions to prevent further infiltration.

Zhang et al. (2006) investigated the failure of the Guohui Dam in China, examining the geometric and hydraulic criteria for internal erosion in gravelly materials [8]. They employed unsaturated-saturated seepage theory to analyze Guohui Dam, as the gravelly materials were unsaturated prior to reservoir water penetration. Water infiltration was simulated through several scenarios, and the results indicated that once reservoir water entered the gravelly materials, a perched water table formed and moved horizontally within the gravel layers, exiting through the downstream slope. The hydraulic gradient of this perched water was greatest near the saturated zone, creating conditions conducive to internal erosion and piping failure. This study demonstrated that the dam's geometric and hydraulic conditions can significantly influence the occurrence of internal erosion and dam failure. Athani et al. (2015) analyzed seepage and stability results for an earth dam using the finite element method [9]. They concluded that increasing the Young's modulus of the core and shell reduces the maximum settlement of the dam crest, while changes in the internal friction angle are crucial in meeting stability criteria. Sazzad et al. (2015) examined seepage through earth dam bodies from a structural safety perspective [10]. They used numerical and analytical analyses to evaluate the effects of factors such as the shape and size of the mesh, the shape of the internal clay core, and the upstream and downstream slopes, as well as

foundation permeability, on seepage rates. Various geometric models of dams were developed, and the discharge rate was analyzed for each model. The findings showed that mesh shape and size have a negligible effect on seepage. When a clay core is present, neither the upstream nor the downstream slope affects seepage. Moreover, without a clay core, seepage is independent of foundation permeability; however, in the presence of a clay core, foundation type (permeable or impermeable) impacts seepage when hydraulic conductivity is high. Tan et al. (2017) used Monte Carlo simulation and random field theory to examine the impact of spatial variations in hydraulic parameters on seepage in earth dams [11]. They applied the van Genuchten model for the soil-water characteristic curve (SWCC) and treated the soil characteristic curve parameters α, n, and the saturated hydraulic conductivity (k_s) as random variables with a lognormal distribution. Results revealed that the coefficient of variation (COV) for n and k_s exerts a stronger influence on seepage rates, whereas the COV for α is less significant. Furthermore, increasing the horizontal correlation length leads to higher mean seepage rates and greater variability. This study showed that disregarding the spatial variation of hydraulic parameters can lead to conservative estimates in earth dam design. Wang et al. (2023) investigated piping failure in tailings dams [12]. They examined the effect of fine particles on the piping process by combining permeability tests and a discrete particle flow model. The results showed that increasing the number of fine particles reduces the permeability coefficient and raises the critical water height. Additionally, changes in pore ratio and particle size distribution were significantly affected by fine particles. From a meso-structural standpoint, this study also offered new approaches to preventing piping and water infiltration failures in tailings dams. Liu et al. (2024) explored internal erosion failures in sliding dams with highly permeable regions [13]. They employed physical models to analyze how the position and particle size distribution of these zones affect water seepage. The results indicated that piping and downstream slope erosion are among the most critical failure modes for this type of dam. Moreover, increasing the permeability coefficient intensifies the speed and severity of piping, slope erosion, and dam collapse.

A review of previous research reveals that there have been relatively few studies on estimating the risk of internal erosion. The limited existing research primarily adopts a deterministic approach and focuses on estimating seepage flow. The main novelty of this study lies in the integration of transient seepage analysis with probabilistic internal erosion assessment, enabling a more realistic evaluation of erosion risk in earth dams under uncertain hydraulic conditions. In addition, this work applies and compares the Rosenblueth and Harr point-estimate methods to examine their capability in estimating the probability of internal erosion. A further contribution is the development of an efficient computational framework for internal erosion risk analysis in earth dams, explicitly accounting for the uncertainty associated with permeability.

2. Materials and Methods

2.1. Seepage Analysis in the Dam Body and Foundation

Generally, the total seepage discharge through a dam is divided into two components: seepage through the foundation and seepage through the dam body. In other words:

Total Seepage = Seepage Through Foundation + Seepage Through Body

In this study, the finite element method (FEM) using SEEP/W software was applied to calculate the seepage flow from the dam body and foundation. Seepage in earth dams can be analyzed under two conditions: steady-state and transient. Steady-state

conditions occur when neither the seepage discharge nor the water head at any point changes over time, which typically happens a few years after the dam reservoir is filled. In a water-saturated soil medium, the governing equation for steady-state (i.e., time-invariant) seepage flow is the Laplace equation, expressed as:

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0 \quad (1)$$

Where h is the water head, dependent on the spatial coordinates x and y. The terms k_x and k_y represent the permeability coefficients of the soil in the horizontal and vertical directions, respectively. By contrast, transient (unsteady) flow occurs when the water head at each point changes over time. For instance, once an earth dam is completely filled, the water head in its core rises until it reaches the free water surface [14].

It should be noted that as the reservoir level changes due to filling or drawdown, the water level in the dam's core also fluctuates. In high dams, such transient flow conditions may warrant further analysis. Under transient conditions, the governing equation (the Boussinesq equation) is:

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = c \frac{\partial h}{\partial t} \quad (2)$$

Here, $\frac{\partial h}{\partial t}$ indicates that the water head changes over time, thereby signifying unsteady flow. Various approaches exist for solving the Boussinesq equation; however, with advances in computing technology over the past three decades, numerical analyses using specialized software have largely replaced analytical methods.

2.2. Uncertainty Analysis Using the Rosenblueth Point Estimate Method

The Rosenblueth Point Estimate Method is a commonly used approach in uncertainty analysis and risk assessment for estimating statistical parameters of complex functions. In this method, the probability distribution of random variables is approximated by a limited number of representative points (often two points for each variable). By combining these points, an estimate of the target expected values (such as mean and variance) is obtained. The primary advantage of this method lies in its significant reduction in computational cost compared to Monte Carlo simulations or Latin Hypercube Sampling (LHS), while still maintaining adequate accuracy in assessing the impact of uncertainties [15]. The permeability coefficients of the materials used in the dam inherently carry uncertainty. To consider this uncertainty in the piping risk analysis, it is necessary to run a transient (non-steady-state) model of water seepage through the dam and foundation. Since this model is very time-consuming to run, using a Monte Carlo simulation for analyzing uncertainty in soil permeability coefficients was not feasible. Consequently, the Rosenblueth Point Estimate Method and the Harr Point Estimate Method—both of which entail far fewer computations—were employed for this purpose. This section provides a detailed explanation of the Rosenblueth method.

Considering a model $W = g(X)$, which has a random variable X and its three initial moments or probability density function (PDF) are known, the Rosenbluth probability point method estimates the original probability density function of the random variable X by assuming that the entire probability mass of X is concentrated in two points x_- and x_+ .

Using the two-point estimation, the positions x_- and x_+ and their corresponding probability masses p_- and p_+ are determined in such a way that the first three moments of the random variable X are preserved. In the general case where a model has N dependent random variables, the m^{th} moment of the model $W = g(X_1, X_2, \dots, X_N)$ around the origin can be estimated as follows:

$$E(W^m) \approx \sum p_{(\delta_1,\delta_2,\ldots,\delta_N)}\left[w_{(\delta_1,\delta_2,\ldots,\delta_N)}\right]^m \quad (3)$$

where the index δ_i indicates the sign and can only be + or – and indicates that the random variable X_i has the value $x_{i+} = \mu_i + x'_{i+}\sigma_i$ or $x_{i-} = \mu_i + x'_{i-}\sigma_i$. The probability mass at each 2^N point, $p_{(\delta_1,\delta_2,\ldots,\delta_N)}$, can be estimated as follows:

$$p_{(\delta_1,\delta_2,\ldots,\delta_N)} = \prod_{i=1}^{N} p_i\sigma_i + \sum_{i=1}^{N-1}\left(\sum_{j=i+1}^{N}\delta_i\delta_j a_{ij}\right) \quad (4)$$

$$a_{ij} = \frac{\rho_{ij}/2^N}{\sqrt{\prod_{i=1}^{N}\left[1+\left(\gamma_i/2\right)^2\right]}} \quad (5)$$

3.2. Uncertainty Analysis Using the Harr Point Estimate Method

The Harr Point Estimate Method is an approximate technique for uncertainty assessment in which a limited number of specific points in the space of random variables are used to estimate the statistical behavior of a system. Rather than performing extensive sampling (such as Monte Carlo simulations), this method selects particular points in the probabilistic space that represent the mean and variance of the random variables. The response function is then evaluated at these points, and the resulting values are combined to estimate statistical parameters of complex functions. Because only a small number of points are required and the computational demands are relatively low, the Harr method is especially effective and efficient in applications involving computationally intensive numerical models (such as finite element models) [16].

For a multivariate model $W = g(X_1, X_2, \ldots, X_N)$ with N random variables, the Haar method selects the calculation points at the intersection of the axes of the N eigenvectors with the plane of a multidimensional sphere of radius $\sqrt{N}$ in the eigenspace as follows:

$$x_i = \mu \pm \sqrt{ND^{1/2}}\vartheta_i \ , \quad i = 1.2.\ldots,,.N \quad (6)$$

Where $x_{i\pm}$ represents the coordinate vector of N random variables in the parameter space corresponding to the i[th] eigenvector, ϑ_i and $\mu = (\mu_1, \mu_2, \ldots, \mu_N)^t$ is the vector of means of N random variables X. Based on the 2N points determined by equation (6), the function values at each 2N point can be calculated. Therefore, the m[th] output moment of the model W about the origin can be calculated based on the following equations:

$$\overline{w_i^m} = \frac{w_{i+}^m + w_{i-}^m}{2} = \frac{g^m(x_{i+}) + g^m(x_{i-})}{2}, \quad i = 1,2,\ldots..,N\ ;\ m = 1,2,\ldots \quad (7)$$

$$E[W^m] = \mu_m(W) = \frac{\sum_{i=1}^{N}\lambda_i\overline{w_i^m}}{\sum_{i=1}^{N}\lambda_i} \ , \quad m = 1.2.\ldots \quad (8)$$

2.4. Risk Analysis of Internal Erosion

Exceeding the critical hydraulic gradient triggers internal erosion. Piping is one of the most significant mechanisms of hydraulic failure in earth dams and happens when the hydraulic gradient (i) surpasses its critical value (i_{cr}). This phenomenon leads to internal erosion and ultimately creates preferential flow paths within the dam body or foundation. In this study, numerical and probabilistic methods were used to analyze this phenomenon. To evaluate the dam's stability against internal erosion (piping), the hydraulic gradient must first be calculated, which is defined as follows:

$$i = \frac{\Delta h}{L} \quad (9)$$

where Δh denotes the head difference and L is the length of the flow path through the soil. The seepage path length is estimated using Equation (10), which is presented as follows:

$$L = \sqrt{\Delta h^2 + \Delta x^2} \quad (10)$$

where Δh is the change in piezometric head between two points (in meters) and Δx is the horizontal distance between those two points (in meters). Therefore, the probability of internal erosion is given by:

$$R = P(i > i_{cr}) \quad (11)$$

Since the hydraulic gradient i depends on the soil permeability K, and K itself carries uncertainty, this uncertainty propagates to the risk of internal erosion. Given the uncertainty in K, the hydraulic gradient i is assumed to follow a normal distribution. Similarly, based on the minimum and maximum critical gradient values from reputable geotechnical references, the critical gradient i_{cr} is also assumed to follow a normal distribution. Thus, for the probability that $i \geq i_{cr}$ one can write:

$$P(i > i_{cr}) = P(i - i_{cr} > 0) \quad (12)$$

Since i and i_{cr} are both normally distributed, their difference $(i - i_{cr})$ is also normally distributed. Therefore, the variable Z is standard normally distributed:

$$Z = \frac{(i - i_{cr}) - \mu_{i-i_{cr}}}{\sigma_{i-i_{cr}}} \quad (13)$$

where $\mu_{i-i_{cr}}$ is the mean of $(\mu_{i-i_{cr}})$ and $\sigma_{i-i_{cr}}$ is the standard deviation of the normally distributed variable $(i - i_{cr})$. According to the standard normal distribution:

$$\mu_{i-i_{cr}} = \mu_i - \mu_{i_{cr}} \quad (14)$$

$$\sigma_{i-i_{cr}} = \sqrt{\sigma_i^2 + \sigma_{i_{cr}}^2} \quad (15)$$

Therefore:

$$P(i > i_{cr}) = \Phi\left(\frac{\mu_{i-i_{cr}}}{\sigma_{i-i_{cr}}}\right) \quad (16)$$

Where $\Phi\left(\frac{\mu_{i-i_{cr}}}{\sigma_{i-i_{cr}}}\right)$ is the cumulative distribution function (CDF) of the standard normal distribution, whose values can be obtained from standard normal distribution tables.

2.4. Case Study

Lar Dam is one of the major dams in Iran, located on the southern slopes of Mount Damavand (Figure 1), approximately 84 kilometers northeast of Tehran. This earth dam, with a clay core, has been under construction since the 1971s to supply drinking water to Tehran and control seasonal floods, and it was commissioned in the 1981s. Lar Dam has a considerable reservoir capacity, meeting part of Tehran's drinking water needs and providing the potential for hydropower generation. However, reports indicate seepage issues in the dam body and foundation, which could be related to internal erosion [17], [18]. Thus, assessing the risk of internal erosion in Lar Dam is particularly critical given its strategic importance for water resource management in metropolitan Tehran. Lar Dam is an earth-fill dam with a height of 105 meters from the riverbed and a crest length of 1,300 meters. Table 1 presents the permeability characteristics of Lar Dam, and Figure 2 illustrates the zones of materials used in the dam body and foundation.

Table 1. Permeability Coefficients of Materials Used in Lar Dam (Moshanir Company, 2022)

Zone	Material Type	Average Permeability (m/day)	Average Permeability (cm/s)	Minimum Permeability (m/day)	Minimum Permeability (cm/s)	Maximum Permeability (m/day)	Maximum Permeability (cm/s)
1A, 1B	Silty Clay	0.0006912	7.966×10^{-7}	0.0005184	5.998×10^{-7}	0.000864	1×10^{-6}
2	Sandy Silt	0.001728	1.999×10^{-6}	0.000864	1×10^{-6}	0.002592	3×10^{-6}
3A, 3B, 3C, 4, 5	Sandy Gravel	475.2	0.5498	86.4	0.1	864	1
Foundation	Silty Clay	0.0004752	5.498×10^{-7}	0.0000864	1×10^{-7}	0.000864	1×10^{-6}

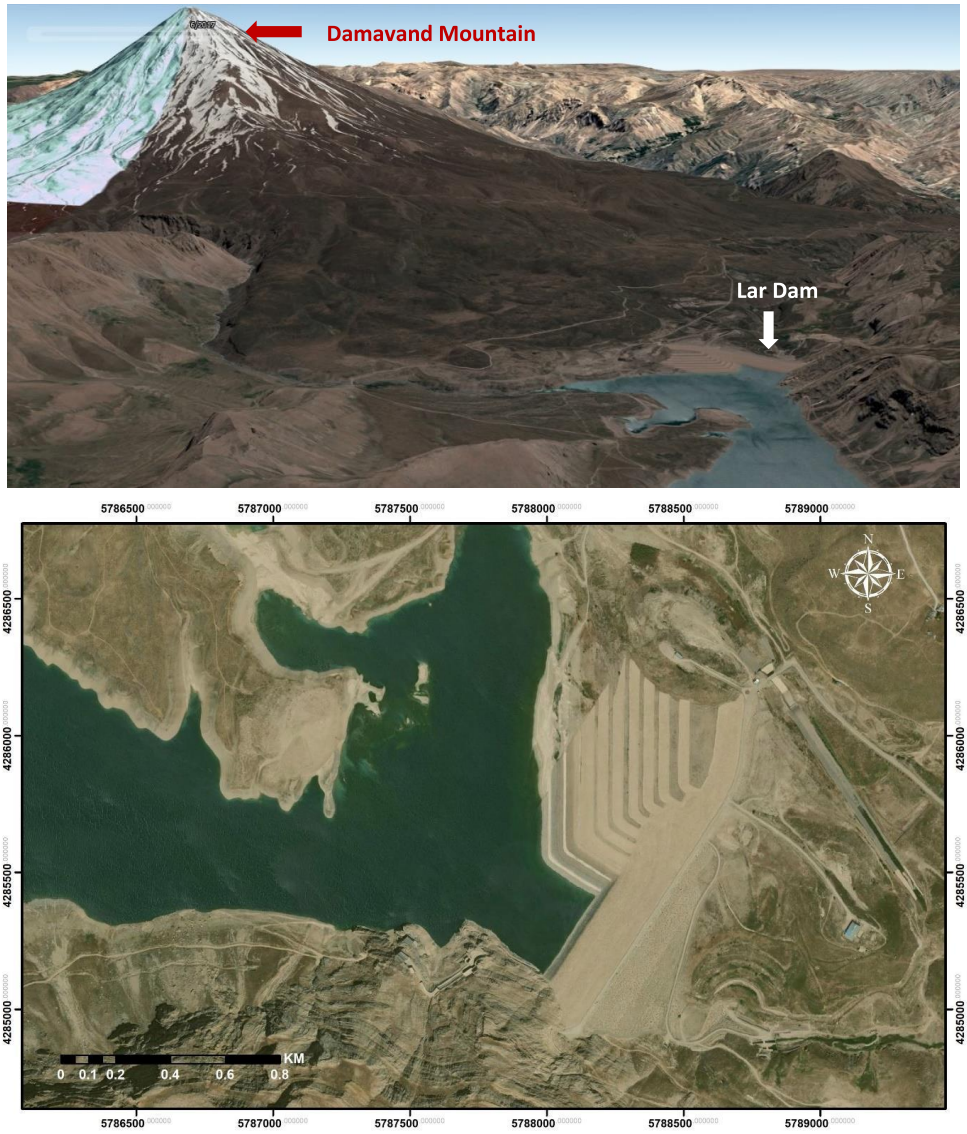


Figure 1. (a) The location of Lar Dam relative to Mount Damavand; (b) Layout of Lar Dam

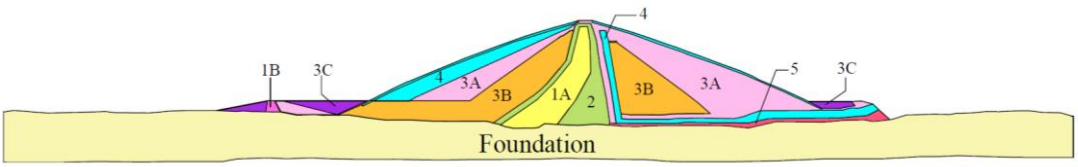


Figure 2. Zones of Materials Used in Lar Dam (Moshaniro Company, 2023).

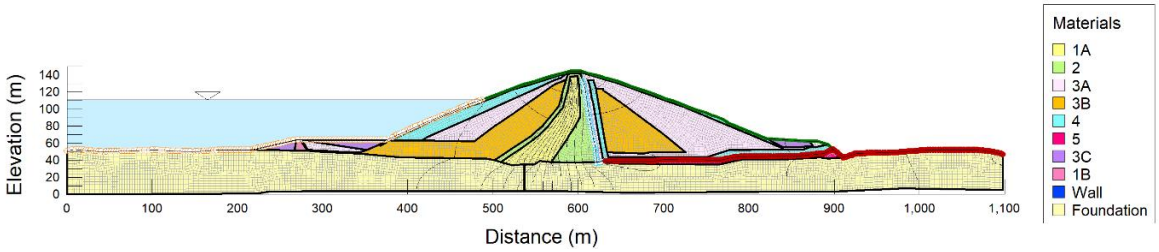


Figure 3. Finite Element Mesh and Materials Used in Lar Dam.

3. Model Implementation and Results Analysis

In this study, the SEEP/W software was used to numerically solve the seepage equations using the finite element method. In the finite element approach, both triangular and quadrilateral elements were employed to construct the computational mesh. A sensitivity analysis was conducted on the mesh dimensions, and ultimately a mesh element size of 3 meters was adopted. Figure 3 shows the computational mesh of the dam body and foundation, along with its zoning. It is worth noting that the longitudinal profile shown corresponds to Chainage 320 on the right bank of Lar Dam.

For the seepage analysis in this software, after constructing the geometry of Lar Dam and assigning its permeability coefficients equal to their average values—based on the type of materials used and the data received from the Tehran Regional Water Company (Table 1)—the model was executed under both steady-state and unsteady-state conditions.

Under steady-state conditions, when the reservoir water level was at its historical maximum, the seepage discharge in the critical zone of the core was estimated to be 0.12 m^3/day, and when the reservoir water level was at its historical minimum, it was estimated to be 0.036 m^3/day.

The boundary conditions applied in the steady-state analysis were as follows:

- Upstream boundary: Head (m)
- Filter boundary: Total Flux (m^3/day)
- Downstream boundary: Pressure Head (m)

For the upstream boundary, the water head was set equal to the maximum inflow head into the dam. The Total Flux and Pressure Head at the filter and downstream boundaries were set to zero. The numerical results of the steady-state seepage analysis are presented in Figure 4(a).

The boundary conditions for the unsteady-state analysis were similar to those of the steady-state, with the exception that in the unsteady-state case, the upstream boundary condition is defined as a time-dependent Head (m) function due to variations in the reservoir water level over time. In the Lar Dam model, this function was defined based on daily water level data collected over the period from 2001 to 2023 (corresponding to 8159 days). The results of the seepage analysis over these 8159 days are shown in Figure 4(b).

As Figure 4(b) illustrates, the highest hydraulic gradient occurs in the core zone of the dam. Therefore, in the calculation of the internal erosion risk, the hydraulic gradient in the dam core was used, since the other zones or transition areas exhibit more favorable hydraulic gradient conditions compared to the core.

Validation of the Seepage Model for the Dam Body and Foundation

After running the model, the piezometric head outputs were obtained. To validate the model, the observed data for the only available piezometric head measurement at location H38 (shown in Figure 5) was received from the Tehran Regional Water Company, and the model results were compared with these measured values. It is noteworthy that the model results were obtained using the average permeability coefficients. Figure 6 presents a comparison between the modeled and measured results. As observed visually, the model has successfully predicted both the temporal variations and the magnitude of the piezometric head. It should also be mentioned that although additional piezometers have been installed in the dam body, data from these devices were unfortunately not available, which limited further evaluation of the model's performance.

Analysis of the Probability of Failure Due to Internal Erosion

The execution time for the transient analysis on a system with a Core™ i5-12450H processor and 16 GB of memory was around 2.5 hours. This duration is considerable and limits the use of random sampling methods such as the Monte Carlo method and Latin Hypercube Sampling. Therefore, in this study, two-point estimate methods—the Rosenblueth Point Estimate Method and the Harr Point Estimate Method—were utilized.

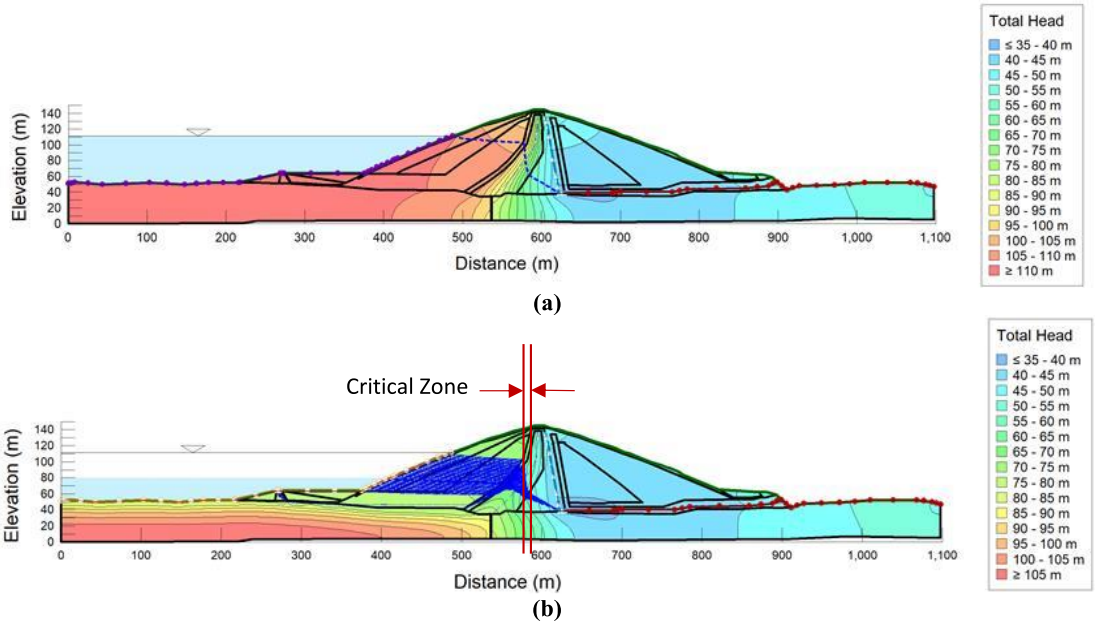


Figure 4. Leakage from the Lar Dam Body in Case (a) Steady Analysis and (b) Unsteady Analysis.

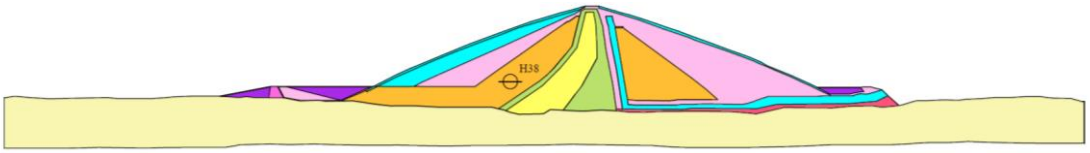


Figure 5. Location of Piezometer H38 on the Dam Body.

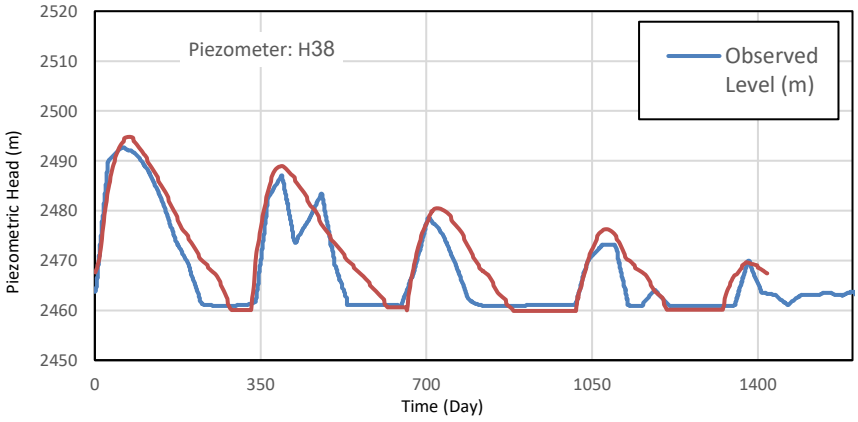


Figure 6. Changes in Piezometric Head from 2019 to 2023.

Rosenblueth Point Estimate Method

Considering that the dam body and foundation of Lar Dam consist of four different types of materials, and that the permeability coefficients of these materials inherently possess uncertainty, the uncertainty of the permeability coefficient for each material was modeled using a normal distribution. The mean of each distribution was taken as the average value reported in Table 1, and the standard deviation of the probability distribution was estimated as $\sigma \approx \frac{K_{max} - K_{min}}{6}$ based on the theoretical principles of probability and statistics. This choice is justified by the fact that, for a normal distribution, approximately 97.5% of the data lie within ±3σ. According to the theory of the Rosenblueth Point Estimate Method, if the probability of internal erosion is represented by a function $g(k)$ and the permeability coefficient k is treated as the uncertain input parameter, then for Lar Dam there are N=4 random input variables corresponding to the permeability coefficients. Consequently, according to the Rosenblueth method, $2^4 = 16$ possible points must be evaluated. Assuming a normal probability mass function (PMF) for the permeability variable, the PMF for the random variable K is approximated at two points, K_- and K_+. Since the probability distribution of the random variable K is symmetric (i.e., normally distributed), the skewness factor is γ=0. In this case, according to the relations in the Rosenblueth method, we have: $k'_- = k'_+ = 1$ and $p_- = p_+ = 0.5$. If the permeability coefficients for the core, transitional zone, shell, and foundation are denoted by K_1, K_2, K_3 and K_4 respectively, the objective is to calculate the probability of internal erosion in the critical zone of the core. Now, it is necessary to evaluate the function $g(k)$, which represents the probability of internal erosion (Pp), at the 16 computational points (i.e., $2^4 = 16$) as determined by the Rosenblueth method, as listed in Table 2. Subsequently, based on these probability values, the expected value of the internal erosion probability (μ = E(P_p)) and its standard deviation (σ_{P_p}) were calculated. The values for the probability of internal erosion occurrence at these points are provided in Table 3.

According to the formulas, assuming that the permeability coefficients are independent, the probability mass (P) is obtained as 0.625. Then, by calculating $P_p \times P$ and $(P_p^2) \times P$ and summing the resulting values, we obtain μ_{P_p}= E(P_p) = 0.225 and E(P_p^2) = 0.066. Therefore, the expected value of the "internal erosion probability" and its standard deviation, as determined by the Rosenbluth method, are 0.225 and 0.1, respectively.

Table 2. Permeability Coefficients Calculated Using the Rosenbluth Method Formulas.

Point	K_1 (m/day)	K_2 (m/day)	K_3 (m/day)	K_4 (m/day)
1	0.00072	0.00216	540	0.00054
2	0.00072	0.00216	410.4	0.0004104
3	0.00072	0.00216	540	0.0004104
4	0.00072	0.00216	410.4	0.00054
5	0.00072	0.001296	540	0.0004104
6	0.00072	0.001296	410.4	0.00054
7	0.00072	0.001296	540	0.00054
8	0.00072	0.001296	410.4	0.0004104
9	0.0006624	0.00216	540	0.00054
10	0.0006624	0.00216	410.4	0.0004104
11	0.0006624	0.00216	540	0.0004104
12	0.0006624	0.00216	410.4	0.00054
13	0.0006624	0.001296	540	0.0004104
14	0.0006624	0.001296	410.4	0.00054
15	0.0006624	0.001296	540	0.00054
16	0.0006624	0.001296	410.4	0.0004104

Table 3. Probability of Internal Erosion Occurrence at Estimation Points Using the Rosenbluth Method.

Point	K_1 (m/day)	K_2 (m/day)	K_3 (m/day)	K_4 (m/day)	Probability of Internal Erosion Occurrence (P_p)
1	+	+	+	+	0.114
2	+	+	-	-	0.115
3	+	+	+	-	0.115
4	+	+	-	+	0.108
5	+	-	+	-	0.396
6	+	-	-	+	0.357
7	+	-	+	+	0.357
8	+	-	-	-	0.369
9	-	+	+	+	0.084
10	-	+	-	-	0.086
11	-	+	+	-	0.09
12	-	+	-	+	0.09
13	-	-	+	-	0.342
14	-	-	-	+	0.323
15	-	-	+	+	0.335
16	-	-	-	-	0.342

Harr's Point Estimation Method

Considering four different types of materials for the Lar dam, and based on Harr's probability point estimation theory, if the permeability coefficient of the materials is assumed to be a non-deterministic parameter, there are N = 4 random variables. According to Harr's method, a total of 2×N = 8 possible points must be examined. The points evaluated using Harr's method, calculated according to the existing formulas, are presented in Table 4. Additionally, the values for the probability of internal erosion occurrence at these points are provided in Table 5.

Table 4. Permeability Coefficients Calculated at the Evaluation Points Using Harr's Method.

Point	K_1 (m/day)	K_2 (m/day)	K_3 (m/day)	K_4 (m/day)
+1	0.0007488	0.0017	475.2	0.0004752
-1	0.000633	0.0017	475.2	0.0004752
+2	0.0006912	0.0026	475.2	0.0004752
-2	0.0006912	0.000864	475.2	0.0004752
+3	0.0006912	0.0017	604.8	0.0004752
-3	0.0006912	0.0017	345.6	0.0004752
+4	0.0006912	0.0017	475.2	0.0006048
-4	0.0006912	0.0017	475.2	0.0003456

Table 5. Calculation of the Internal Erosion Occurrence Probability at the Evaluation Points Using Harr's Method.

Point	Probability of Internal Erosion (P_p)	$(P_p)^2$	$\overline{P_p}$	$\overline{P_p^2}$
+1	0.258	0.066	0.156	0.034
-1	0.055	0.003		
+2	0.204	0.041	0.202	0.041
-2	0.2	0.04		
+3	0.173	0.03	0.275	0.086
-3	0.377	0.142		
+4	0.204	0.041	0.206	0.042
-4	0.207	0.043		

According to the formulas in Harr's method, the mean value is μ_{P_p} = 0.21 and $E(P_p^2)$ = 0.05. Therefore, the expected value of "internal erosion occurrence" and its standard deviation using Harr's method are 0.21 and 0.08, respectively, which are values close to those obtained from the Rosenbluth probability point method.

The probabilistic analyses performed using the Rosenblueth and Harr point-estimation methods yielded closely comparable results, indicating a mean internal erosion probability of approximately 0.225 and 0.21, respectively, with standard deviations of about 0.10 and 0.08. This close agreement is important because it suggests that the estimated erosion risk is not strongly dependent on the specific point-estimation scheme adopted, despite the different sampling strategies of the two methods. The Rosenblueth method evaluates 2^N=16 combinations for the four uncertain permeability variables, whereas Harr's method requires only 2N=8 evaluations. Therefore, the similarity of the results implies that Harr's method can provide a computationally efficient approximation of internal erosion risk for practical engineering applications, while the Rosenblueth method offers a somewhat richer exploration of the uncertainty space. From an engineering perspective, the fact that both methods produce a moderate probability of internal erosion, on the order of 0.21 to 0.23, indicates that the risk cannot be considered negligible and should be explicitly accounted for in seepage safety assessments of the dam.

A closer examination of the Rosenblueth evaluation points shows that the predicted probability of internal erosion varies noticeably across different combinations of permeability values. In particular, the lowest probabilities are associated with cases in which the permeability of the transition zone is relatively high, whereas markedly larger probabilities are observed when the transition-zone permeability takes its lower value. This pattern suggests that the transition zone plays a critical role in controlling the hydraulic response of the dam and, consequently, the susceptibility of the core to internal erosion. Physically, a reduction in the permeability of the transition zone may hinder the proper dissipation or redistribution of seepage flow, thereby increasing unfavorable hydraulic gradients in the critical part of the core. By contrast, variations in other material zones also affect the results, but their influence appears to be less dominant than that of the transition zone. This observation is valuable because it identifies a parameter to which the predicted erosion response is particularly sensitive, and it can help prioritize site investigation, material characterization, and quality control during design or rehabilitation.

The dispersion of the estimated probabilities, reflected by standard deviations in the range of 0.08 to 0.1, also deserves attention. These values show that uncertainty in permeability does not merely shift the mean estimate, but also creates a relatively broad spread in possible erosion outcomes. In other words, even if the mean

probability is moderate, there are combinations of material properties for which the likelihood of internal erosion becomes substantially higher. This finding highlights the importance of incorporating parameter uncertainty into seepage and erosion assessments, rather than relying solely on deterministic permeability values. It also supports the use of probabilistic frameworks in dam safety evaluation, because such approaches are able to capture both the expected level of risk and its variability. Overall, the results demonstrate that uncertainty in hydraulic properties, especially permeability, has a meaningful impact on the predicted internal erosion potential and should therefore be considered an integral part of risk-informed assessment for earth dams.

4. Conclusion

In this study, the risk of internal erosion for the Lar dam was calculated using the Rosenbluth and Har uncertainty analysis methods, considering the uncertainty in the permeability coefficients of the materials. The main findings are summarized as follows:

1. The results obtained from both Rosenblueth and Harr point estimate methods indicated that the probability of internal erosion is 0.22 and 0.21, respectively, suggesting a relatively high-risk level for the dam. This result is consistent with recent studies by consulting engineers regarding the safety of this dam.
2. The calculated probabilities of internal erosion occurrence from both the Rosenbluth and Harr methods are very similar, and their standard deviations are nearly equal, confirming the accuracy of the computations.
3. The findings also highlight that uncertainty in permeability coefficients plays a significant role in the variability of hydraulic gradients and, consequently, in the initiation of internal erosion. Higher permeability values, especially in coarse-grained materials, can lead to increased hydraulic gradients, thereby increasing the possibility of internal erosion. This emphasizes the importance of accurately characterizing material properties in dam safety assessments.
4. The standard deviation values obtained by both methods are approximately 0.10, which reflects the range of variability in the internal erosion probability.
5. In the present study, the computational burden of the Rosenbluth method is twice that of the Harr method, even though both methods produced identical results. Generally, the computational effort for the Rosenbluth method is 2N and for the Harr probability point method is also 2N. However, despite the lower computational load of the Harr method, it is only applicable to symmetric probability distributions. If the probability distribution of the variables is asymmetric, the asymmetry is not considered in the Harr method, whereas the Rosenbluth method accounts for the skewness of the random variables in the risk analysis. In this study, the use of a normal probability distribution, which has central symmetry, resulted in identical outcomes for both methods.
6. The close agreement between the results of the two methods demonstrates the reliability of point-estimate techniques in probabilistic risk assessment. In particular, the Harr method, due to its lower computational cost, can be considered an efficient alternative to the Rosenblueth method

in cases where input variables follow symmetric distributions.

For future research on this topic, the following recommendations are proposed:

1- Investigate the risk of internal erosion using the Monte Carlo method by incorporating the uncertainty in the critical hydraulic gradient.
2- Consider the uncertainty of other parameters, such as the specific weight of the materials, in the stability analysis.
3- Conduct a stability analysis of the Lar dam that accounts for seismic dynamic loads and incorporates the uncertainty of the dam's behavioral model parameters.
4- Evaluate the risk of liquefaction for earth dams.

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